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Processes of Constructıon of Transformatıonal Geometry Knowledge of Secondary School Students: Lesson Plan Sample

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Abstract: The goals of the new global education system and the expectations placed on students have been shaped to ensure that individuals can succeed in a rapidly changing world, and countries have developed various reforms and strategies to integrate 21st century skills into the education system. Abstraction is a thinking process that enables individuals to derive general principles or concepts from specific information and experiences. This process is crucial for the development of 21st century skills such as problem solving, critical thinking and creative thinking. In particular, the RBC+C abstraction model stands out as an effective tool for deepening mathematical thinking. The RBC+C model helps students to develop abstract thinking skills while showing how these skills can be used in practical applications.

The RBC+C abstraction model is an important building block in transformation geometry lesson plans because it provides an effective approach to deepening students' understanding of geometric transformations. This study consists of daily lesson plans created to gain an in-depth understanding of how middle school students construct transformation geometry knowledge, the difficulties they encounter in the process of constructing knowledge, the paths they follow in the learning process, and their abstraction processes. The implementation of the daily lesson plans created in the study by the teachers will allow an in-depth examination of the processes of secondary school students' construction of transformation geometry knowledge, the paths and experiences they follow in the learning process, the difficulties they encounter in the process of knowledge construction, and the abstraction process to be carried out more effectively, and therefore the subject of transformation geometry or the concepts of reflection and translation in mathematics teaching will provide faster learning.
Keywords: Lesson plan, transformation geometry, RBC+C model

1. Introduction

The aims of the new global education system and its expectations of students are designed to ensure that individuals can succeed in a rapidly changing world. This system focuses not only on academic knowledge but also on 21st century skills such as critical thinking, creativity, digital literacy, collaboration, communication, cultural awareness, lifelong learning and information and media literacy (Binkley et al., 2012).

Critical thinking enables students to deepen their knowledge and find creative solutions to various problems. Paul and Elder (2008) state that critical thinking helps individuals to look at information from different perspectives and draw logical conclusions. In addition, problem solving skills increase students' ability to cope with complex situations (Facione, 2011). Creativity refers to students' ability to develop new and original ideas, while innovation is the practical application of these ideas. Digital literacy is the ability of students to use digital technologies effectively and to acquire knowledge and skills related to these technologies. Jones and Martz (2007) emphasise that digital literacy is essential for students to be successful in the digital age. Collaboration and communication skills enable students to work effectively in groups and to express their thoughts clearly. In their study, Johnson and Johnson (1999) found that collaboration and effective communication improve group dynamics and the learning process.

Global citizenship and cultural awareness help students to understand and develop solutions to social, economic and environmental problems around the world and instil values such as international cooperation and empathy (Miller, 2021). Schattle (2008) states that global and cultural awareness broadens an individual's worldview and provides a necessary foundation for international cooperation. Lifelong learning refers to the continuous development of students' knowledge and skills beyond the educational process. Candy (2002) highlights the positive impact of lifelong learning on the personal and professional development of individuals.

Information and media literacy is the ability of students to access and evaluate information and to analyse media content. The article "The Importance of Media Literacy in the 21st Century" states that information and media literacy is essential for individuals to gain the ability to access and critically evaluate information effectively in the digital age (Smith, 2022). Integrating these skills into education facilitates students' adaptation to rapidly changing technological and economic conditions and provides them with the necessary skills to solve complex problems (Jones, 2023).

Different countries around the world have developed different reforms and strategies to integrate 21st century skills into the education system. Turkey is one of them. With the 2017 Education Curriculum (2017) and the Turkish Century Education Model (2024) announced by the Ministry of National Education of the Republic of Turkey (MoNE), the curriculum has focused on more student-centred learning and applied learning methods. These curriculum changes are structured to support 21st century skills and provide students with opportunities for creative thinking and problem solving (Yüksel, 2019). In addition, these programmes organised by the MoNE are designed to support teachers' digital literacy, creative teaching methods and critical thinking skills (Akbaba, 2020). Modern school buildings, flexible learning spaces and technology-equipped classrooms provide a suitable environment for students to develop 21st century skills (Aydın, 2018). Organisations such as the Educational Research and Application Centres (EDUCATION-AR) continue to work to ensure that the Turkish education system provides students with 21st century skills and adapts to global education standards, as well as researching and implementing innovative methods in education (Özdemir, 2019).

Abstraction is a thinking process that enables individuals to derive general principles or concepts from specific information and experiences. This process is critical to the development of 21st century skills such as problem solving, critical thinking and creative thinking. Abstraction increases the ability to solve complex problems by transforming knowledge into general principles (Papert, 1980). This skill enables individuals to deal with new and different situations by generalising from specific situations and data. This allows individuals to gain flexibility and creativity in problem solving processes. Another important aspect of abstraction is that it supports critical thinking skills. Şönfeld et al. (2009) emphasised that abstract thinking improves students' ability to analyse and solve complex problems. Abstraction enables individuals to structure knowledge, understand relationships and evaluate different perspectives. This process helps individuals to approach knowledge more deeply and to develop complex thinking skills. Göksu and Çakır (2017), while explaining how abstract thinking skills are supported in modern education systems and how these skills improve students' performance, emphasised that education systems should be designed to develop students' abstract thinking skills.

In mathematics education, abstraction refers to the treatment of mathematical concepts and processes in a general, abstract way. Tall (2004) stresses the importance of abstraction in the mathematical learning process and argues that as students gain the ability to work more effectively with abstract concepts, their mathematical thinking will deepen. Moses and Cobb (2001) found that abstraction improves students' problem solving skills and helps them to learn mathematical concepts in a more general and systematic way.

The concept of abstraction can be interpreted from two distinct perspectives: cognitive and sociocultural (Yeşildere, 2006). One of the theories that interprets abstraction from a sociocultural perspective is the RBC+C abstraction model. This model is formed by using the initials of the epistemic acts of recognising (R), using (B), creating (C) and reinforcing (+C). The RBC+C model was developed by Hershkowitz, Schwarz, and Dreyfus (2001) to examine mathematical abstraction processes by incorporating the epistemic act of reinforcement into the RBC (recognition-use-construction) abstraction model, which was subsequently expanded by Dreyfus in 2007. The RBC+C model is based on the observation of cognitive actions, with the mental processes of the participants defined in relation to these actions.

The RBC+C model facilitates the observation of the structures formed in the process (Dreyfus, 2007; Dreyfus & Tsamir, 2004; Tsamir & Dreyfus, 2002). In addition, the RBC+C abstraction model provides individuals with a deeper understanding of mathematical concepts, enables the development of mathematical teaching strategies, the cognitive aspects of abstraction contribute to the development of students' problem solving skills, make mathematics education more inclusive and meaningful, and help students to solve mathematical problems more effectively (Ruzsa, 2020; Boudon, 2019; Coyne, 2021; Collins, 2022).

In the literature, studies investigating the effectiveness of the RBC and RBC+C models have examined the processes of knowledge construction through problem situations given to one or more students on a particular unit (Tsamir & Dreyfus, 2002; Dreyfus & Tsamir, 2004; Özmantar & Monaghan, 2005; Yeşildere, 2006; Dreyfus, Hadas, Hershkowitz, & Schwarz, 2006; Schwarz & Dreyfus, 2009; Kaplan & Açıl, 2015; Ulaş & Yenilmez, 2017; Bulut, 2018; Kobak-Demir & Gür, 2019; Temiz, 2019; Eldekçi, 2019; Eroğlu, 2021; Bütüner & İpek, 2023). A review of international studies on abstraction found a limited number of studies (Altun & Yılmaz, 2010; Güler & Arslan, 2018) that examined students' abstraction of coordinate analytical geometry, and thus abstraction of coordinate system-related topics. In their study, Altun and Yılmaz (2010) examined high school students' construction and consolidation processes of piecewise function knowledge and observed that students used previously constructed knowledge in their subsequent work and correctly constructed and consolidated piecewise function knowledge at a certain level. Güler and Arslan (2018) investigated the process of constructing the concept of rotation, which is a structure related to analytical geometry, and concluded that pre-service teachers were deficient in using the formula for rotation transformation in the plane, tended to memorise, and could not construct the formulas in question.

RBC+C theory can be used to examine the processes of constructing and reinforcing mathematical structures from secondary school to graduate school. For this reason, the process of constructing middle school students' knowledge of transformation geometry was presented in this study by creating an example lesson plan. Transformation geometry is included in the middle school mathematics curriculum at the eighth grade level (MEB, 2017; Turkey Century Education Model, 2024). However, only the concepts of translation and reflection are included in the programme. Formulating formulas for reflection and translation requires mathematical abstraction. It is believed that this study will contribute to the studies in mathematics education by examining this process in depth within the framework of a certain learning theory, in order to eliminate the problems experienced in learning mathematics.

2. Daily Lesson Plan for the Process of Creating Transformational Geometry Knowledge

2.1 I. Preparation :

Course Title:	Mathematics
Unit Title:	Transformation Geometry
Topic Title:	Reflection and Translation
Duration:	1 lesson hour
Strategy:	Discovery Learning
Resources and Tools:	Activity sheet, pen, notebook
Goals:	To perform the epistemic action of recognition regarding the concept of reflection and translation
Target Behaviors:	1) To recognize the image of points, line segments and other shapes as a result of reflection. 2) To recognize the image of polygons as a result of translation and reflection.

2.2 I. Lesson Delivery:

To make sure the students are aware of the subject, the teacher says: "Today we will start with transformation geometry. Distribute the Activity Sheet in Appendix 1 to the students. "We will learn the topic with the story of Fairytale Land and the Pattern Wizard. Let's read the story," and the lesson begins. Attention is drawn to the expressions "shifting in a certain direction and distance, i.e. translation" and "mirroring a shape around an axis, i.e. reflection" in the story. Pupils are asked to find examples of translation and reflection in the classroom, in their belongings or in nature. The teacher gives the necessary feedback and corrections to the students' answers. The activity sheet in Appendix 2a is distributed to the pupils. The story is continued until the first step instruction. After the teacher has read the instructions, students are given 10 minutes to study the visuals and write answers (Appendix 2b). At the end of the time, the students are asked to say what they wrote for the first visual (Appendix 2c). In order to create an interactive classroom environment where students listen to each other and discuss their ideas, students in the listener position are asked for their ideas about what they have written. The teacher only listens without commenting. After all the students' writings have been read in turn, students can make additions to what they have written if they wish. After the oral answers, the teacher gives hints or guidance to all the pupils to correct their mistakes if they have made mistakes. In this way the teacher increases the students' self-confidence. If none of the students can reach the correct answer, the teacher gives the correct answer. The teacher repeats the definition of reflection and translation and reinforces the concepts by showing different examples. The same process is repeated for the other picture and the activity is finished.

In order to make it easier for teachers to determine whether their students' verbal and written expressions represent the act of recognition during the implementation of the lesson plan, the key expressions that define the epistemic act of recognition are shown in Figure 1, using the RBC+C model.

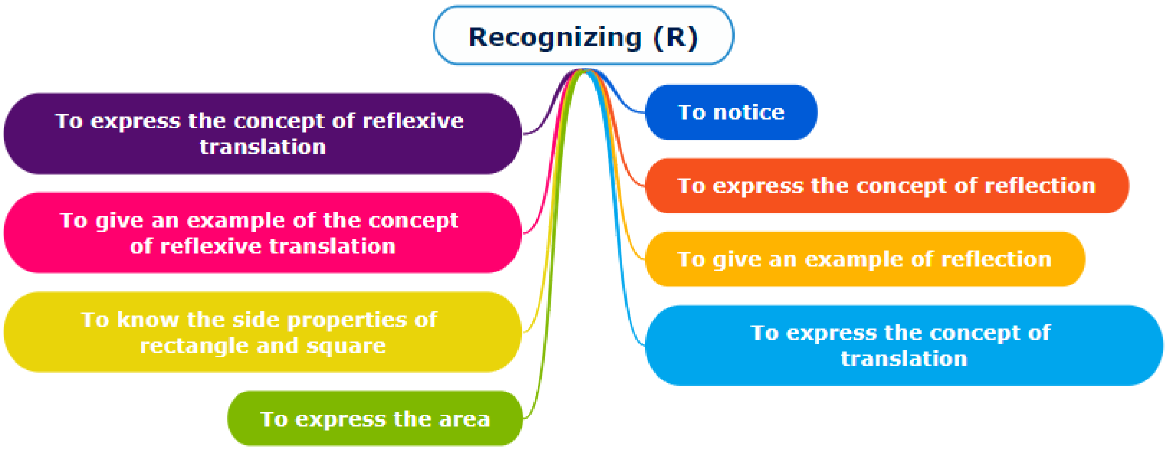

Figure 1: Key Phrases Defining the Epistemic Act of Recognition

2.3 II. Preparation:

Course Title:	Mathematics
Unit Title:	Transformation Geometry
Topic Title:	Reflection and Translation
Duration:	1 lesson hour
Strategy:	Discovery Learning
Resources and Tools:	Activity papers, colored pencils, notebook
Goals:	To perform the epistemic action of bulding with the concept of reflection and translation
Target Behaviors:	1)To create the image of points, straight lines and other shapes as a result of reflection. 2) To create the image of polygons as a result of translation and reflection.

2.4 II. Lesson Delivery:

The teacher draws attention to the lesson by saying "What did we do last lesson?" and the students are made to remember the concepts of reflection and translation. Appendix 3a activity sheet is distributed to the students. Then, the teacher says, "In this lesson, we will continue the story of transformation geometry and also the story of Fairy Tale Land and Pattern Wizard. Let's start reading the second part." After the story, the directive is read by the teacher (Appendix 3b). Students are given 12 minutes to make individual drawings on the activity page in accordance with the instruction (Appendix 3c). After the time, the students are asked to show how they drew the translation in accordance with the first item of the instruction (Appendix 3d). Students are given 2 minutes to reach a conclusion by discussing the accuracy of their answers. Thus, by supporting peer learning, students are encouraged to comprehend the subject in more depth, improve their communication skills, develop their critical thinking skills and increase their self-confidence. At the end of the time, after the teacher shows the correct answer, the teacher provides clear guidance on what the student has done correctly and what he/she needs to improve, and offers the student the opportunity to correct his/her mistakes and discover the correct ways of learning. The students are given 2 minutes each to discuss their answers to the second and third items of the instruction and reach a conclusion. At the end of the time, after the teacher shows the correct answer, the teacher again provides clear guidance on what the student has done correctly and what he/she needs to improve.

The students are provided with the activity sheet set out in Appendix 4a. The third part of the narrative is then conveyed to a selected student (Appendix 4b). Prior to presenting the instructions, the instructor states, "The subsequent objective is to engage in a reflective exercise," and proceeds to read the instructions aloud. Students are permitted a period of 12 minutes to create individual drawings in accordance with the instructions provided on the activity sheet (Appendix 4c). Subsequently, the students are required to demonstrate how they have drawn the reflection pattern in accordance with the initial item of the instruction. They are permitted a period of two minutes to conclude their responses by discussing the accuracy of their answers. At the conclusion of this period, the teacher presents the correct answer and provides clear guidance on the student's successful responses and areas for improvement. The activity is then concluded by applying the same process steps to the remaining items of the instruction.

In order to make it easier for teachers to determine whether their students' verbal and written expressions represent the act of using during the implementation of the lesson plan, the key expressions that define the epistemic act of using using the RBC+C model are shown in Figure 2.

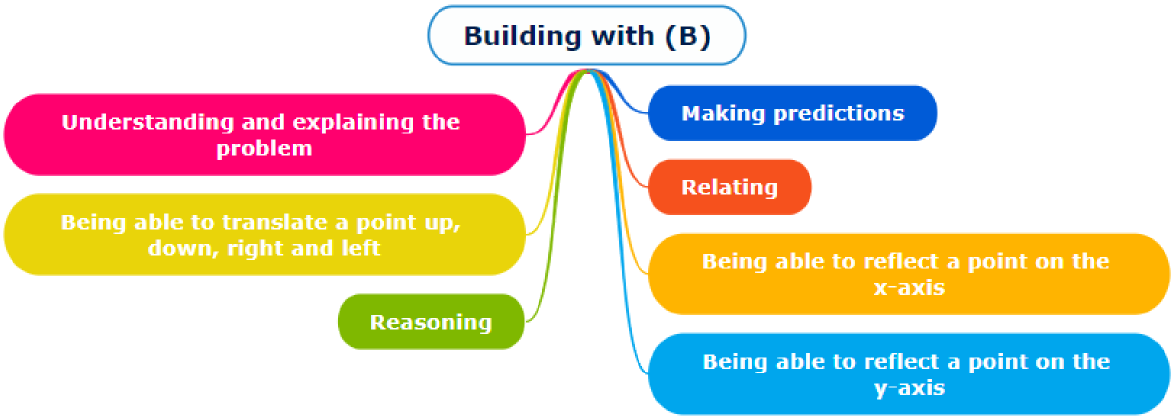

Figure 2: Key Phrases Defining the Epistemic Act of Building with

2.5 III. Preparation :

Course Title:	Mathematics
Unit Title:	Transformation Geometry
Topic Title:	Reflection and Translation
Duration:	1 lesson hour
Strategy:	Discovery Learning
Resources and Tools:	Activity papers, colored pencils, notebook
Goals:	To perform the epistemic action of constructing related to the concept of displacement
Target Behaviors:	To create patterns, motifs and similar visuals and construct an original carpet pattern as a result of displacement and reflections.

2.7 III. Lesson Delivery:

The plan of the activity and the diary (Preparation III and IV) for the epistemic act of constructing has been designed to be carried out in two consecutive lessons.

The instructor stated that the class would proceed with both transformation geometry and the narrative concurrently. At the outset of our narrative, the definitions of translation and reflection were elucidated. "Who would like to provide a brief overview of the previous lesson's content?" he begins the class by refreshing the students' recollection of the preceding lesson's material. Following the definitions, students are required to provide an example of reflection and translation. The teacher provides prompt feedback and assists the students in identifying and rectifying any shortcomings in their understanding. The students are provided with the Appendix 5a activity sheet. The text at the commencement of the activity is read by a selected student. The students are provided with ten minutes to identify the coordinates of fourteen points on the pattern and record them in the designated section of the table, titled "Initial State Coordinate of the Point." (Appendix 5b) At the conclusion of the allotted time, the instructor informs the students that they will perform the requisite translations in the table by employing the coordinates of the points on the pattern(Appendix 5c). Therefore, they should record the coordinates they ascertain in the designated column. The students are permitted a period of fifteen minutes to complete the table. The students are required to provide a comprehensive response to the question, "How did the coordinates change?" In the event that the students require further clarification, an illustrative example may be provided for a point that is not included in the table (Appendix 5c). Following a 15-minute interval, the students are prompted to provide the final coordinates of the points. It is explained that if the students want to change their answers, they can correct them with a different coloured pencil or fill in the blanks. The teacher guides the pupils to the correct answer by allowing them to compare their answers one by one and by giving hints and instructions. Effective communication in the classroom helps students to recognise the change in the abscissa and ordinate values as a result of moving up, down, right and left in the coordinate system.

In the second question, "If the coordinate (-5,8) obtained by translating 3br upwards and 4br to the left is the initial coordinate of point R", the students are asked to perform the reverse operation of the operations they applied in the table (Appendix 5d). The reverse operation helps the students to better understand the logic of translation and the order of the operations.

In the question "How can you formulate the translation of a point selected in regions I, II, III and IV of the coordinate system?", the teacher tells the students that they can benefit from the table of the first question and the pattern given at the beginning (Appendix 5e). The teacher asks the students to formulate a formula by making a generalisation specific to the region chosen, based on this one point, not on any point in the regions. In the process of knowledge construction, the teacher should guide the students with hints and involve them in the learning process without directing them. Pupils' translation formulae are listened to in turn and clear information is given about what the pupil has done correctly and what he/she needs to improve.

2.8 IV. Preparation:

Course Title:	Mathematics
Unit Title:	Transformation Geometry
Topic Title:	Reflection and Translation
Duration:	1 lesson hour
Strategy:	Discovery Learning
Resources and Tools:	Activity papers, colored pencils, notebook
Goals:	To perform the epistemic action of constructing related to the concept of reflection
Target Behaviors:	The objective is to create patterns, motifs and similar visuals and construct an original carpet pattern as a result of displacements and reflections.

2.9 IV. Lesson Delivery:

The students are provided with the Appendix 6a activity sheet. The students are required to complete the table provided in response to the fourth question within a 15-minute timeframe, utilising the coordinates of the points delineated on the pattern (Appendix 6b). It is emphasised that students should provide a comprehensive response to the question "How did their coordinates change?" (Appendix 6b) In the event that the students require further clarification, the teacher provides an illustrative example pertaining to a point that is not included in the table. Following a 15-minute period, the instructor requests that the students provide the final coordinates of the points. The instructor permits students who wish to alter their responses to do so with a different coloured pencil or by filling in the blank spaces. The teacher facilitates the students' attainment of the correct answer by enabling them to compare the responses one by one and by providing guidance and direction.

In the fifth and sixth questions, "If the coordinate of point T formed by the reflection of point T with respect to the x-axis is (-5,8), what is the initial coordinate of point T?" and "If the coordinate of point T formed by the reflection of point T with respect to the y-axis is (-5,8), what is the initial coordinate of point T?" respectively, students are asked to perform the inverse operation of the operations in the table (Appendix 6c; Appendix 6d). The inverse operation allows the students to understand the logic of reflection on the x and y axes.

In the question "How can you formulate the reflection of a point selected in the I., II., III. and IV. region in the coordinate system according to the x and y axis?", students are told that they can benefit from the table of the first question and the pattern given at the beginning (Appendix 6e). The students are asked to formulate a formula individually by making a generalisation specific to the selected region based on this one point, not a point in the regions. In this process, the teacher acts as a guide and gives hints to the students when necessary.

In order to make it easier for teachers to determine whether their students' verbal and written expressions represent the act of construction when implementing the lesson plan, the key expressions that define the epistemic act of construction are given in Figure 3 using the RBC+C model.

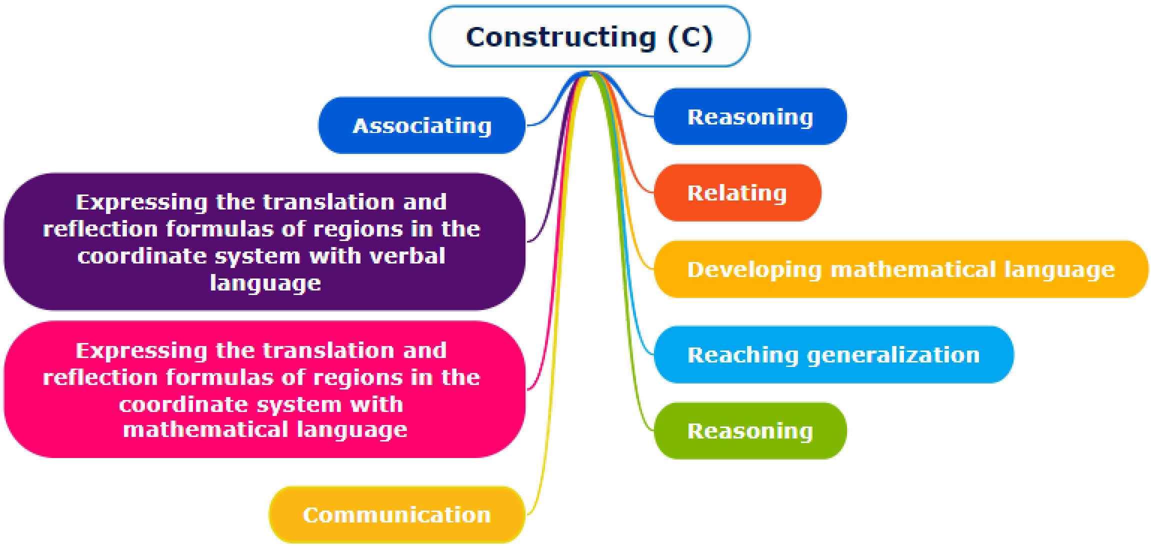

Figure 3: Key Phrases Defining the Epistemic Act of Constructing

2.10 V. Preparation:

Course Title:	Mathematics
Unit Title:	Transformation Geometry
Topic Title:	Reflection and Translation
Duration:	1 lesson hour
Strategy:	Discovery Learning
Resources and Tools:	Activity papers, colored pencils, notebook
Goals:	To perform the epistemic action of consoladation regarding the concept of reflection and displacement
Target Behaviors:	The objective is to create cities, home decorations, clothes and paintings using the knowledge of displacement and reflection of patterns and motifs.

2.11 V. Lesson Delivery:

The lesson commences with a query regarding the methodology employed in translating a point selected from the I., II., III. and IV. regions of the coordinate system, as addressed in the preceding lesson. Subsequently, the students are prompted to recall their prior knowledge by being asked to describe the manner in which they formulated the reflection formula in accordance with the x and y axes. The instructor informs the students that the current lesson will conclude today, marking the end of the transformation geometry unit. Subsequently, the students are provided with the Appendix 7a activity sheet. One student is selected at random, and the narrative and subsequent instructions are read aloud (Appendix 7b). Students are allotted a period of ten minutes for the initial question (Appendix 7c). The students are then required to indicate whether the visuals in question demonstrate the presence of translation, reflection, or a combination of both. Upon the conclusion of the designated period, the instructor requests that each student provide a response to the initial visual (Appendix 7d). In this process, the teacher's role is limited to that of a passive listener, refraining from providing any form of feedback. Once all responses have been collected, the student(s) who provided the correct answer are invited to elucidate the rationale behind their selection. In the event that no student is able to provide the correct answer, the teacher ensures that the students reach the correct answer through the provision of hints and guidance. This process is repeated for each visual.

In the second question, the teacher asks the students to complete the unprinted parts of the pattern on each carpet using unit squares within 10 minutes (Appendix 7e). When the time is up, the students are asked to compare the patterns they have made. After the teacher has shown the correct pattern, the students who made the correct pattern are asked to help the students who made the wrong pattern to correct the pattern.

In the third question the teacher asks the students to create new coordinates individually within 10 minutes by transforming the points whose coordinates are given (Appendix 7f). At the end of the time, the teacher asks the students to give their answers. The teacher tells the pupils that if they want to change their answers, they can correct them with a different coloured pencil or fill in the blanks. The teacher then asks the pupil(s) who found the correct answer to explain the answer.

In the last question, the teacher asks the students to choose any Turkish motif and draw it on the canvas and to create the best pattern on the canvas within 10 minutes using reflection, translation and reflected translation (Appendix 7g). The teacher tells the students that they can do reflection, translation and reflected translation freely after they have found the Turkish motif through books and internet. At the end of the process, the activity is concluded by voting among the students to choose the best among the canvases.

In order to make it easier for teachers to determine whether their students' verbal and written expressions are reinforcing actions while implementing the lesson plan, key expressions that define reinforcing epistemic actions are given in Figure 4 using the RBC+C model.

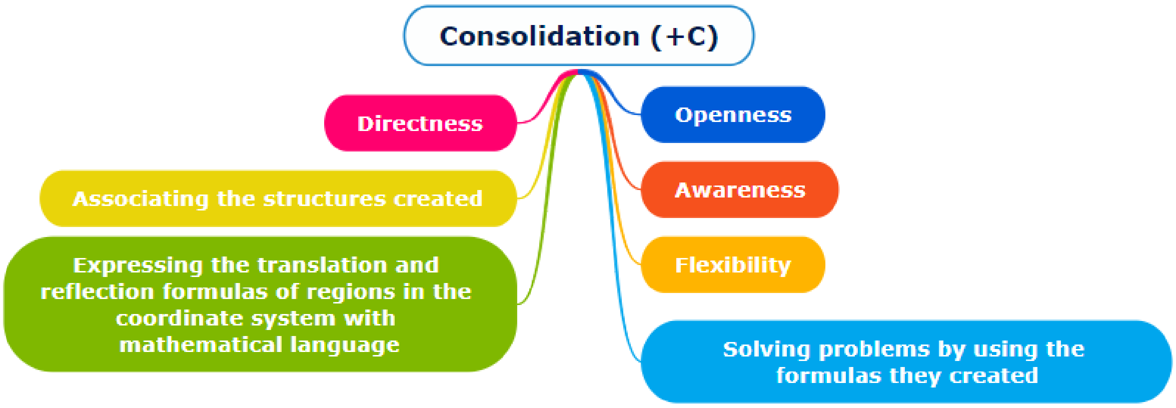

Figure 4: Key Phrases Defining the Epistemic Act of Consolidation

3. Conclusion

As a result, the RBC+C model develops students' in-depth thinking and problem solving skills in mathematics and supports permanent learning (Dreyfus & Tsamir, 2004). It is also an important tool for teachers and educators to monitor learning processes in more detail and develop appropriate strategies for these users (Simpson & Ellison, 2014). The model is an effective technique to enhance the content of mathematics education by structuring comprehensive thinking processes step by step while developing problem solving skills in subjects (Rabardel & Dreyfus, 2009). Therefore, more widespread use of this model in mathematics education and research will positively affect education.

This study consists of daily lesson plans created to get an in-depth idea of how middle school students construct transformation geometry knowledge, the difficulties they encounter in the process of constructing knowledge, the paths they follow in the learning process, and their abstraction processes. The implementation of the daily lesson plans created in the study by the teachers can provide an in-depth examination of the processes of secondary school students' construction of transformation geometry knowledge, the paths and experiences they follow in the learning process, and the difficulties they encounter in the process of knowledge construction, and thus the abstraction process can be carried out more effectively, and thus the subject of transformation geometry or the concepts of reflection and translation in mathematics teaching can be learned more quickly.

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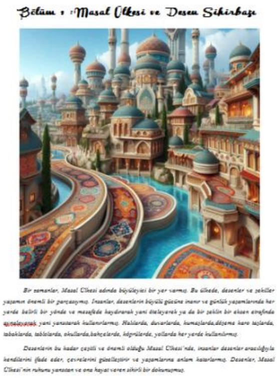
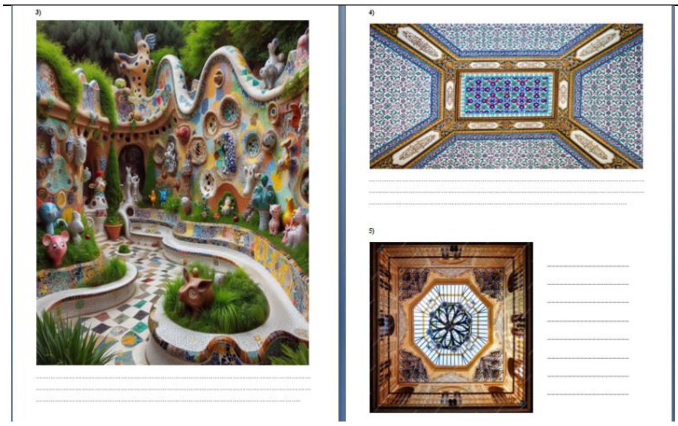
Appendix

Appendix 1:



Appendix 2a:



Appendix 2b:

Yönerge

Sizlerde Masal Ülkesi'nden örnek gösterilen eserleri inceleyip, eserlerin hangilerinde yansıma ve öteleme kullanıldığını detaylı yazın ve anlatın.

Appendix 2c:

1)

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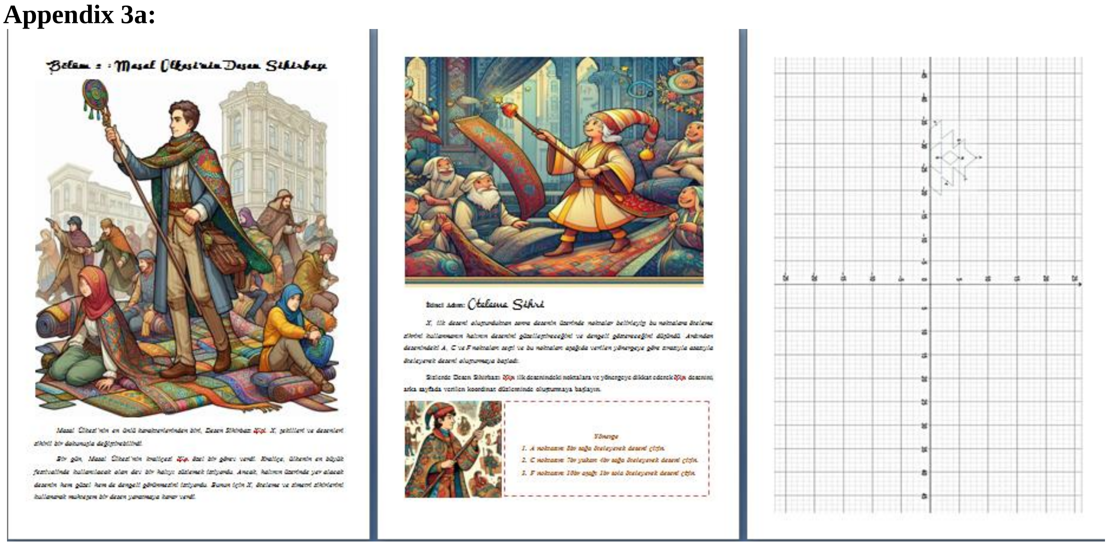
Appendix 3a:



Appendix 3b:

Bölüm 2 : Masal Ülkesi'nin Desen Sihirbazı

Masal Ülkesi'nin en ünlü karakterlerinden biri, Desen Sihirbazı X'di. X, şekilleri ve desenleri sihirli bir dokunuşla değiştirebilirdi.

Bir gün, Masal Ülkesi'nin kraliçesi X'e özel bir görev verdi. Kraliçe, ülkenin en büyük festivalinde kullanılacak olan dev bir halıyı süslemek istiyordu. Ancak, halının üzerinde yer alacak desenin hem güzel hem de dengeli görünmesini istiyordu. Bunun için X, öteleme ve simetri sihirlerini kullanarak muhteşem bir desen yapmaya karar verdi.

Appendix 3c:

İkinci Adım: Öteleme Sihri

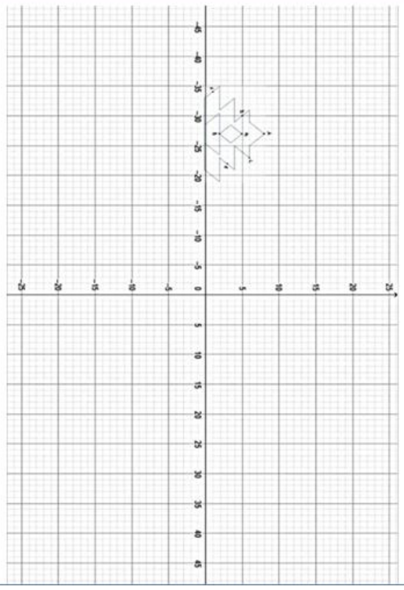
X, ilk deseni oluşturduktan sonra desenin üzerinde noktalar belirleyip bu noktalara öteleme sihrini kullanmanın halının desenini güzelleştireceğini ve dengeli göstereceğini düşündü. Ardından desenindeki A, C ve F noktaları seçti ve bu noktaları aşağıda verilen yönergeye göre sırasıyla asasıyla öteleyerek deseni oluşturmaya başladı.

Sizlerde Desen Sihirbazı X'in ilk desenindeki noktalara ve yönergeye dikkat ederek X'in desenini, arka sayfada verilen koordinat düzleminde oluşturmaya başlayın.

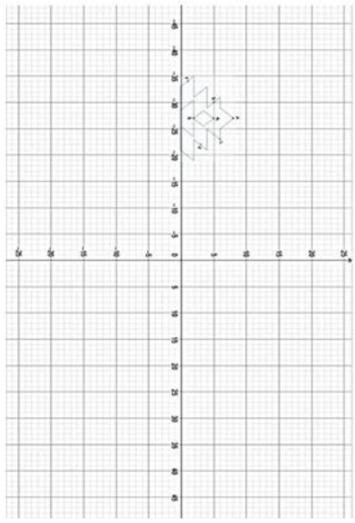
Yönerge

1. A noktasını 8br sağa öteleyerek deseni çizin.
2. C noktasını 7br yukarı 4br sağa öteleyerek deseni çizin.
3. F noktasını 10br aşağı 1br sola öteleyerek deseni çizin.

Appendix 3d:



Appendix 4a:

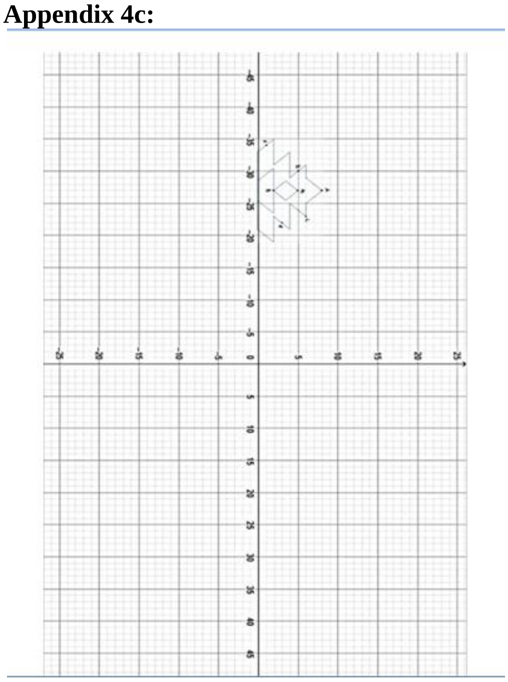


Appendix 4b:

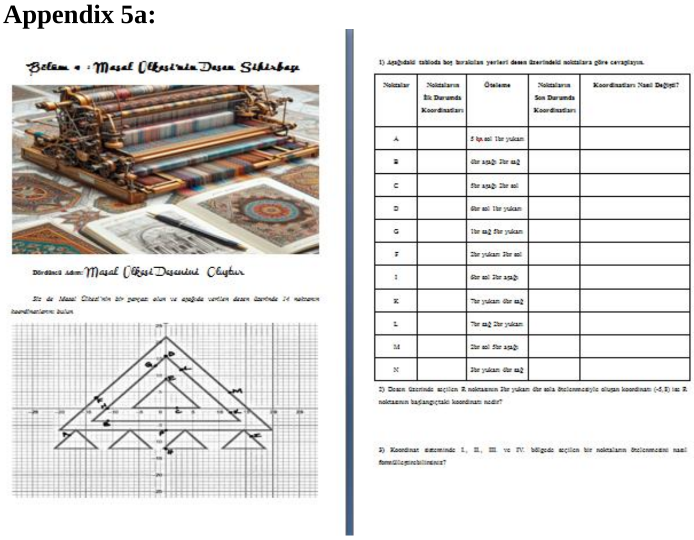
Yönerge

1. *B noktasını x eksenine göre yansımasını alarak diğer noktaları yerleştirip deseni çizin.*
2. *D noktasını y eksenine göre yansımasını alarak diğer noktaları yerleştirip deseni çizin.*
3. *Deseni 8br sağa 12br aşağı öteleyerek oluşan deseni x eksenine göre yansımsını çizin.*
4. *Deseni x eksenine göre yansıtarak oluşan deseni 8br sağa 12br aşağı öteleyerek çizin.*

Appendix 4c:



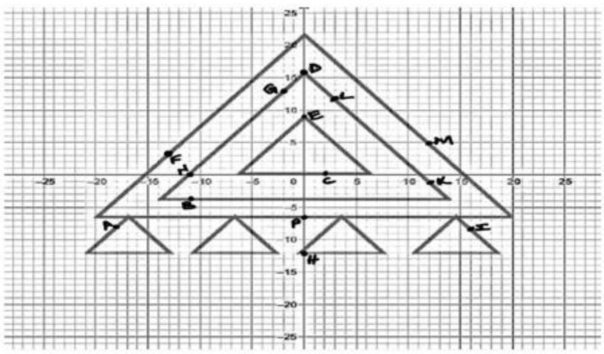
Appendix 5a:



Appendix 5b:

Dördüncü Adım: *Masal Ülkesi Desenini Oluştur*

Siz de Masal Ülkesi'nin bir parçası olun ve aşağıda verilen desen üzerinde 14 noktanın koordinatlarını bulun.



Appendix 5c:

1) Aşağıdaki tabloda boş bırakılan yerleri desen üzerindeki noktalara göre cevaplayın.

Noktalar	Noktaların İlk Durumda Koordinatları	Öteleme	Noktaların Son Durumda Koordinatları	Koordinatları Nasıl Değişti?
A		5 br sol 1br yukarı		
B		4br aşağı 3br sağ		
C		5br aşağı 2br sol		
D		6br sol 1br yukarı		
G		1br sağ 5br yukarı		
F		2br yukarı 3br sol		
I		6br sol 3br aşağı		
K		7br yukarı 4br sağ		
L		7br sağ 2br yukarı		
M		2br sol 5br aşağı		
N		3br yukarı 4br sağ		

Appendix 5d:

2) Desen üzerinde seçilen R noktasının 3br yukarı 4br sola ötelenmesiyle oluşan koordinatı (-5,8) ise R noktasının başlangıçtaki koordinatı nedir?

Appendix 5e:

3) Koordinat sisteminde I., II., III. ve IV. bölgede seçilen bir noktaların ötelenmesini nasıl formülleştirebilirsiniz?

Appendix 6a:

4) Aşağıdaki tabloda boş bırakılan yerleri desen üzerindeki noktalara göre cevaplayın.

Noktalar	Noktaların İlk Durumda Koordinatları	Yansıma	Noktaların Son Durumda Koordinatları	Koordinatları Nasıl Değişti?
A		x eksenine göre yansıması		
		y eksenine göre yansıması		
K		x eksenine göre yansıması		
		y eksenine göre yansıması		
I		x eksenine göre yansıması		
		y eksenine göre yansıması		
L		x eksenine göre yansıması		
		y eksenine göre yansıması		
E		y eksenine göre yansıması		
P		y eksenine göre yansıması		
C		x eksenine göre yansıması		
N		x eksenine göre yansıması		

5) Desen üzerinde seçilen T noktasının x eksenine göre yansımasıyla oluşan koordinatı (-5,8) ise T noktasının başlangıçtaki koordinatı nedir?

6) Desen üzerinde seçilen T noktasının y eksenine göre yansımasıyla oluşan koordinatı (-5,8) ise T noktasının başlangıçtaki koordinatı nedir?

7) Koordinat sisteminde I., II., III. ve IV. bölgede seçilen bir noktaların x ve y eksenine göre yansımalarını nasıl formülleştirebilirsiniz?

Appendix 6b:

4) Aşağıdaki tabloda boş bırakılan yerleri desen üzerindeki noktalara göre cevaplayın.

Noktalar	Noktaların Ilk Durumda Koordinatları	Yansıma	Noktaların Son Durumda Koordinatları	Koordinatları Nasıl Değişti?
A		x eksenine göre yansıması		
		y eksenine göre yansıması		
K		x eksenine göre yansıması		
		y eksenine göre yansıması		
I		x eksenine göre yansıması		
		y eksenine göre yansıması		
L		x eksenine göre yansıması		
		y eksenine göre yansıması		
E		y eksenine göre yansıması		
P		y eksenine göre yansıması		
C		x eksenine göre yansıması		
N		x eksenine göre yansıması		

Appendix 6c:

5) Desen üzerinde seçilen T noktasının x eksenine göre yansımasıyla oluşan koordinatı (-5,8) ise T noktasının başlangıçtaki koordinatı nedir?

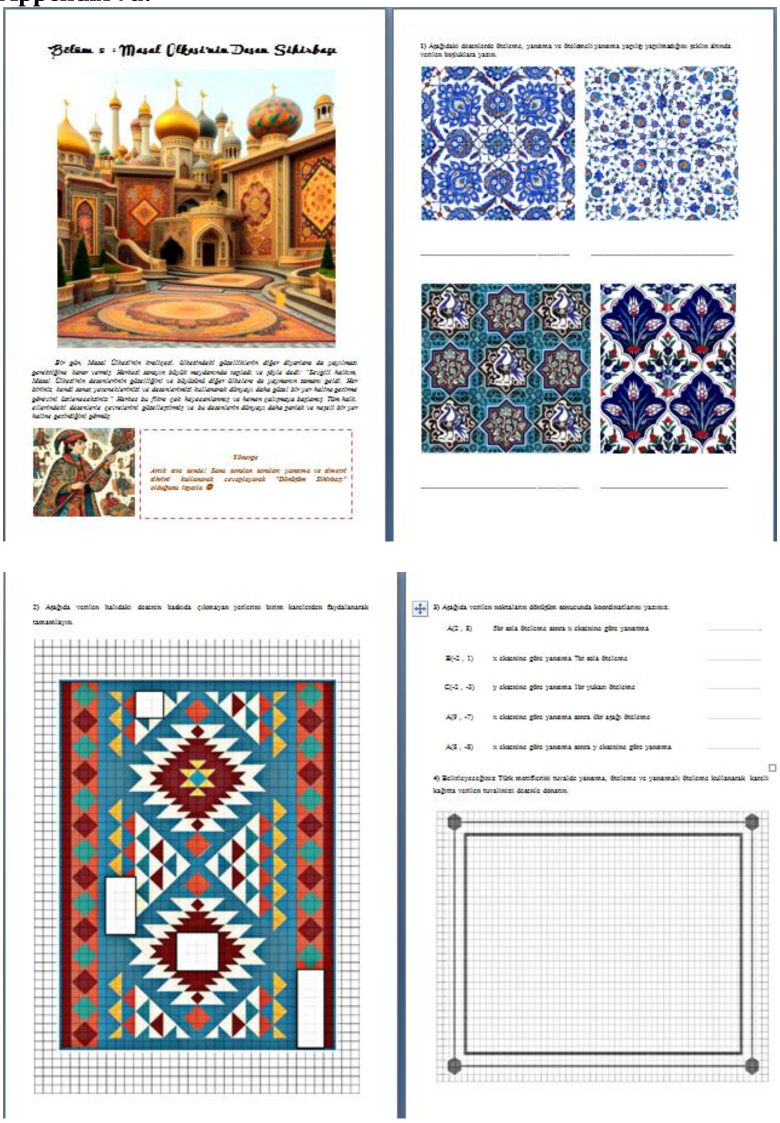
Appendix 6d:

6) Desen üzerinde seçilen T noktasının y eksenine göre yansımasıyla oluşan koordinatı (-5,8) ise T noktasının başlangıçtaki koordinatı nedir?

Appendix 6e:

7) Koordinat sisteminde I., II., III. ve IV. bölgede seçilen bir noktaların x ve y eksenine göre yansımalarını nasıl formülleştirebilirsiniz?

Appendix 7a:



Appendix 7b:

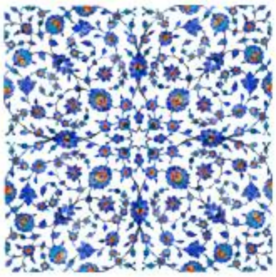
Bir gün, Masal Ülkesi'nin kraliçesi, ülkesindeki güzelliklerin diğer diyarlara da yayılması gerektiğine karar vermiş. Herkesi sarayın büyük meydanında topladı ve şöyle dedi: "Sevgili halkım, Masal Ülkesi'nin desenlerinin güzelliğini ve büyüsünü diğer ülkelere de yaymanın zamanı geldi. Her biriniz, kendi sanat yeteneklerinizi ve desenlerimizi kullanarak dünyayı daha güzel bir yer haline getirme görevini üstleneceksiniz." Herkes bu fikre çok heyecanlanmış ve hemen çalışmaya başlamış. Tüm halk, ellerindeki desenlerle çevrelerini güzelleştirmiş ve bu desenlerin dünyayı daha parlak ve neşeli bir yer haline getirdiğini görmüş.

Yönerge

Artık sıra sende! Sana sorulan soruları yansıma ve simetri sihrini kullanarak cevaplayarak "Dönüşüm Sihirbazı" olduğunu ispatla ☺

Appendix 7c:

1) Aşağıdaki desenlerde öteleme, yansıma ve ötelemeli yansıma yapılıp yapılmadığını şeklin altında verilen boşluklara yazın.



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Appendix 7d:

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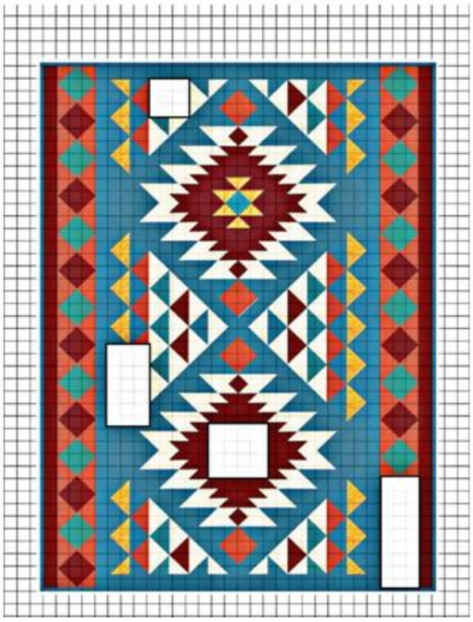
Appendix 7e:

2) Aşağıda verilen halıdaki desenin baskıda çıkmayan yerlerini birim karelerden faydalanarak tamamlayın.



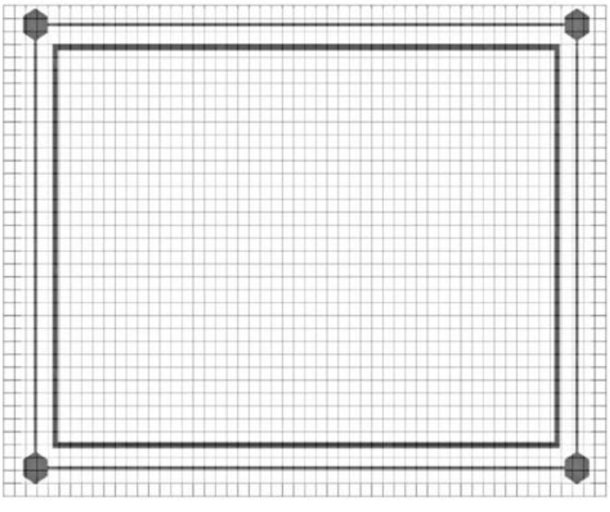
Appendix 7f:

3) Aşağıda verilen noktaların dönüşüm sonucunda koordinatlarını yazınız.

A(2 , 8)	5br sola öteleme sonra x eksenine göre yansıtma	………………….
B(-2 , 1)	x eksenine göre yansıma 7br sola öteleme	…………………
C(-2 , -3)	y eksenine göre yansıma 1br yukarı öteleme	…………………
A(9 , -7)	x eksenine göre yansıma sonra 4br aşağı öteleme	…………………
A(8 , -8)	x eksenine göre yansıma sonra y eksenine göre yansıma	…………………

Appendix 7g:

4) Belirleyeceğiniz Türk motiflerini tuvalde yansıma, öteleme ve yansımalı öteleme kullanarak kareli kağıtta verilen tuvalinizi desenle donatın.



Author Profile

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MEd. Nurgül Bütüner studied Primary Mathematics Education at Istanbul University between 2011-2015, Sociology at Eskişehir Anadolu University between 2019-2022 and Justice at Istanbul University between 2021-2022. She is currently pursuing her Master's degree at Uludağ University, Faculty of Education, Department of Mathematics Education.

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