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Income Distribution in a Developing Market

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Abstract: The shape of income distributions has been represented by several functional forms, including power law, gamma, exponential and even parabolic function. We follow the approach proposed in econophysics for fitting an exponential of Boltzmann- Gibbs to the lower tail of the Colombian income distribution and a Pareto function to the top income segment, and give possible explanations from the perspective of econophysics and economics. We found that, even though the approach holds relatively well in Colombian case, there is an important deviation in the surroundings of the low- middle class level of income, which suggests that other functions might be more accurate.

Keywords: Income distribution, developing market, Boltzmann-Gibbs, Pareto, econophysics.

1. Introduction

According to the Italian economist Vilfredo Pareto (Kotz, S., Kleiber, C, 2003), income distributions have in general a shape resembling what is now called a power law, or Pareto distribution. Let x represents the income, and N the proportion of people earning more than x in a given year. Then, Pareto's law is expressed as

$$\ln N = \ln N_0 - \alpha \ln x, \quad (1)$$

Where N_0 and α are constants. This means that there is a linear relationship between the logarithm of N and the logarithm of x. This distribution is considered heavy-tailed, and, according to Pareto, it could represent a very good fit for virtually any income distribution in the world, empirically speaking. Moreover, since the parameter α is positive, the higher the income level x, the smaller the number N of people with earnings greater than x. This seems to correspond to the situation in real economic systems. However, in later years Pareto became less enthusiastic about this law and stated that it still holds but only in certain, limited conditions. Today, many economists agree that Pareto's law might apply exclusively to the end tails of the income distribution (i.e., people with higher income levels) which would imply that the low-income segment would present another kind of distribution. This is the approach followed in the references Jones, C., Kim, J., (2014) and Acemoglu, D., Robinson, J. (2015).

Some econophysicists have proposed functions that would fit the lower end of the income distribution. In particular, in Dragulescu, A., Yakovenko, V. M.,(2001), it is argued that an exponential curve of the Boltzmann-Gibbs type could be used, based on empirical findings with US and UK data, while Souma, W. (2001), uses data from Japan to fit a log-normal function. Other authors have proposed a gamma function (Angle, J. ,1996).

From the point of view of economy, however, the representation of income distributions by means of fixed fitting functions is not well understood. In fact, only in the case of the power law distribution, it is possible to find the corresponding economic model based upon the inherent patterns of asset accumulation (Acemoglu, D., Robinson, J. 2015), which is widely accepted in the economy community.

In regard to the other distributions (non-Pareto), the state of the art is strongly reduced. As for the Boltzmann-Gibbs exponential distribution, used in Dragulescu, A., Yakovenko, V. M.(2001), a multi-agent model is necessary in order to simulate the transactions that take place in the economy, under the assumption of a constant amount of money in a given period of time and in the presence of debt (credit). The economic explanation of such a behavior is, however, largely missing, and is compared to the behavior of molecules of gas in interaction. Dragulescu and Yakovenko also warn that the Boltzmann-Gibbs distribution is suitable as a fit for the distribution of money (flows of income) rather than the distribution of wealth (stocks of income and assets). The distribution of money would be determined by the equation

$$N(x) = N_0 e^{-\beta x}\ ,\ \beta = \frac{1}{\bar{X}}, \quad (2)$$

Where $\bar{X}$ is the income per capita (per economic agent) and N_0 is a normalizing constant (Dragulescu, A., Yakovenko, V. M., 2000).

Another attempt to find a fitting function for the lower tail of the distribution has been made in Souma, W. (2001), where a log-normal function is used to fit the data for Japan. The fundamental equation is

$$N(x) = \frac{1}{x\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(\log x - \bar{X})}{2\sigma^2}\right] \quad (3)$$

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Where $\sigma 2$ is a variance. Again, the support for this kind of distribution is almost limited to the extrapolation of physical phenomena to the economic world, and economic justifications for its use are not usually put forward in the state of the art (see Montroll, E. W., Shlesinger, M. F. (1983)). In similar grounds are the proposals of Angle, J. (1996) and Shirras, F. (1935) of using a gamma law and a parabolic function as an alternative fit, respectively. Though the latter relies solely on empirical considerations, not on physical analogies, and states that the Pareto law does not hold even among the top earners.

Econophysics (Amarante, V., Jimenez, J. ,2016) tries to find a theoretical explanation for the use of functional representations of the income distribution by using a different approach. An economic system is considered as a highly complex system in which many elements and factors intervene. Due to this complexity, it is not possible to describe the behavior of each element by itself; instead, one considers a set of average quantities which represent the behavior of a large number of elements. It is then assumed that these average quantities satisfy a set of equations that have been established empirically. This approach implies that an economic system may be considered under certain circumstances as a thermodynamic system. If there exists an average quantity that is conserved during a certain period of time, the system can be considered as an equilibrium thermodynamic system; otherwise, it is a non-equilibrium thermodynamic system. Of course, the approach of econophysics is still a subject of intense discussion and no final proof of its applicability has been established so far. Nevertheless, there are many indications that justify its application in realistic economic systems (Dragulescu, A., Yakovenko, V. M., 2001).

In this work, we analyze the income distribution in the case of the Colombian economic system. We follow the approach proposed by Dragulescu and Yakovenko Dragulescu,.(2001) for fitting an exponential of the Boltzmann-Gibbs type to the lower tail of the income distribution and a Pareto function to the top income sector. In addition, we give possible explanations from an economics perspective and from the point of view of the statistical approach as used in econophysics. We find that even though the approach holds relatively well in the Colombian case, there is an important deviation in the surroundings of the low-middle class level of income, suggesting that other functions might be more accurate. We explore different functional dependencies and find that a power law distribution with a different Pareto coefficient matches the income distribution.

2. Fitting the data

For the purposes of this work, we have used the official data used by the Government of Colombia to calculate the country's Gini Index and Lorenz Curve. The data comes from the National Statistics Department (DANE) and corresponds to the Great Integrated Household Survey (GEIH). Without loss of generality, we have considered the data of the last 9 years available for consultation.

The data correspond to the monthly amount of money surveys declared to earn, and as such it might present several forms of bias (Amarante, V., Jimenez, J. , 2016). However, this is the only relatively reliable source of information we have access to for the whole of the population, since taxpayer databases only include the higher percentiles of Colombian earners. Nevertheless, the GEIH data have been used to carry out different analysis that have shown that they correctly represent the real situation of the Colombian economic system.

Data come as a sample at the household level, and include the number of people in the household and the household's per capita income, as well as an expansion factor that tells how many households in the population are represented by each one of the households in the sample (number of households in the sample was 228,662 for 2012). To create the large vector of population we expanded the sample by the expansion factor and the number of people in the household, thus obtaining the vector X, which for 2012 had 45,138,983 observations, a number that roughly corresponds to the population of Colombia for that year. We will use the data and vector for 2012 as a particular example for illustrating in detail the actual situation of the Colombian system.

The country's Lorenz curve for the year 2012 is illustrated in Fig. 1. It clearly shows the structural inequality present in the Colombian economy, as an example of a developing market. The Gini coefficient is considerably high, indicating a high degree of inequality of the income distribution.

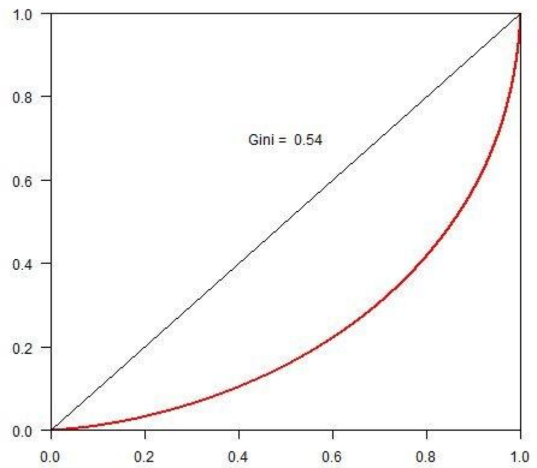

FIG. 1: Lorenz Curve for the year 2012. Own elaboration

When plotted in a log-log scale, the income distribution corresponding to the year 2012 is given in Fig. 2. To improve the statistical result we have considered the entire vector for this year as explained above. The first fact we can notice in this plot is that the distribution is not random, but it clearly shows a structure which resembles the results obtained in econophysics for other economic systems. This shows that the assumptions used in the framework of econophysics can be also applied to the Colombian market which can therefore be considered as a thermodynamic system.

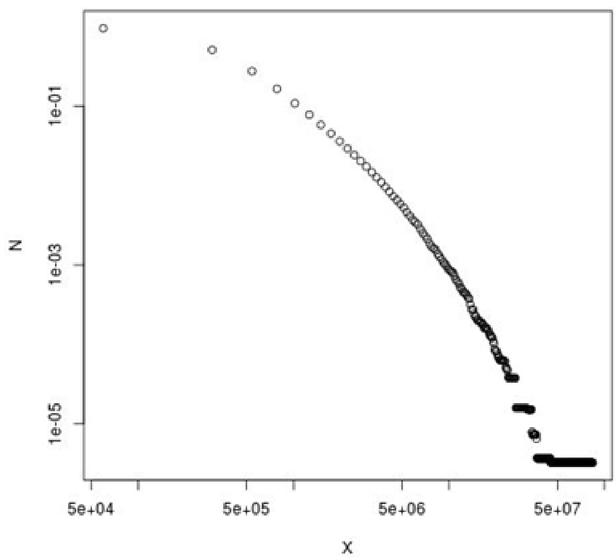

FIG. 2: Income distribution of the Colombian economy for the year 2012 in a log-log scale. Own elaboration.

In Fig. 3, we split the distribution into two parts, following to Dragulescu, A., Yakovenko, V. M. (2000) in the study of the UK and US income distribution, that is, we have an income vector that comprises up to the percentile 95 of the population and another vector for the remaining 5%. We have fitted a Boltzmann-Gibbs exponential function with $\beta = \frac{1}{\bar{X}} = 2.568 \times 10^{-6}$, where

$\bar{X}$ is the mean income of the lower-income part, and a Pareto function to the high-income part, with an estimated α = 2.939. In this plot we have drawn two dashed vertical lines, the one in the left showing the minimum wage[2] and the one in the right representing the x corresponding to the percentile 95, i.e., the income below which 95% of Colombians earn.

[2]Equivalent to constant colombian pesos (COP) 596,398.3 .

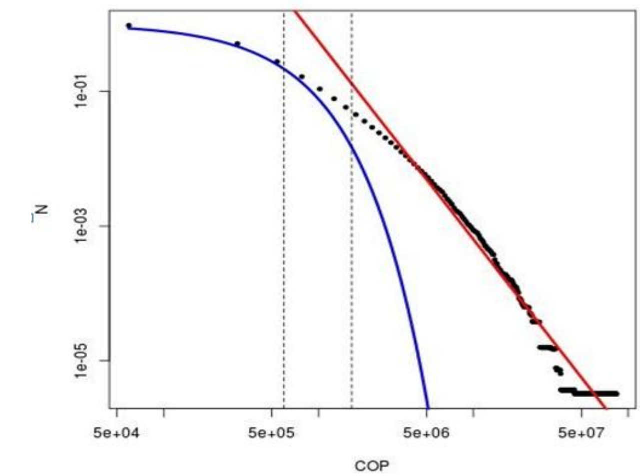

FIG. 3: An exponential function (blue) and a power law (red) fitted to 2012 data. Own elaboration

However, an important part of the observations between these two points is not fitted either by the Boltzmann-Gibbs function or the power law, which indicates that possibly Dragulescu and Yakovenko's hypothesis does not hold completely for the Colombian case. The people contained in this interval are a lower-middle class, whose incomes might be fitted by another power law or even another kind of function like a Gamma or a quadratic function. Moreover, when we examine the histogram of incomes represented in Fig. 4, we can see a peak in the shape of the distribution at around 1/4 the minimum wage (COP150,000).

This could help advance the hypothesis of an even more fragmented income structure in Colombia, which should be modeled using other functional forms.

These findings are in accordance with Quevedo, H., Quevedo, M. N. (2016) where, from an econophysics perspective, it is suggested that the lower tail of the distribution could be fitted by two power laws, given that there is not a single apparent (log-log) linear relationship in the cumulative distribution.

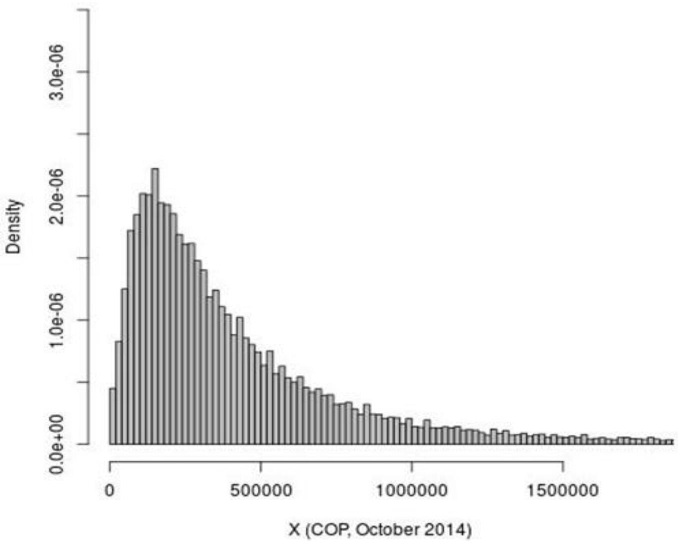

FIG. 4: Distribution of the lowest incomes for the year 2012. Own elaboration

To test the consistency of our results, we have computed for several years the values of the Bolzmann-Gibbs, Pareto and Gini parameters for several years. The results are shown explicitly in Table I. We see that the inequality between high earners is consistently lower than the inequality between low-earners, which might be explained by the large amount of people earning less than the minimum wage. However, it should be kept in mind that the values for percentile 95[3] correspond to a lower-middle class income, which is by no means equatable to the high- incomes of top earners beyond percentile 99.

[3]In the table, COP of October 2014

Year	$\beta \times 10^6$	α	Percentile 95	Gini (total)	Gini (lowinc.)	Gini (highinc.)
2012	3.84	2.93	1,641,753	0.539	0.430	0.292
2013	3.84	2.786	1,316,839	0.539	0.447	0.310
2014	3.74	2.922	1,326,460	0.538	0.437	0.275
2015	3.70	2.799	1,245,632	0.522	0.436	0.292
2016	3.49	2.782	1,298,455	0.517	0.432	0.294
2017	3.13	2.878	1,512,307	0.508	0.453	0.288
2018	2.96	2.837	1,496,048	0.517	0.452	0.295
2019	2.79	2.865	1,599,311	0.526	0.445	0.293
2020	2.65	2.860	1,642,112	0.544	0.433	0.292
2021	2.56	2.939	1,632,812	0.563	0.430	0.282

TABLE 1: Parameters for the years 2012-2021

The parameters show little variation throughout the years, which supports the thesis of the stationary distribution held by Acemoglu, D., Robinson, J. (2015). However, further research must be done in order to identify alternatives to the Boltzmann-Gibbs and Pareto framework outlined here, such as gamma, log-normal or parabolic functions.

3. Econophysics perspective

The results of the last section show that the econophysical conditions are valid in the case of the Colombian economic system. In particular, this implies that the approach of statistical thermodynamics is valid as well Quevedo, H., Quevedo, M. N., (2011). To see this, consider a hypothetical economic system for which a quantity, say M, is conserved; this could be, for instance, the total amount of money in the system which during a certain period of time can be considered as conserved. Suppose that the system is composed of N agents which compete to acquire a participation m of M. Since M is conserved, the system can be considered as characterized by an equilibrium probability distribution of m which is given by the Boltzmann-Gibbs distribution $\rho \sim e{-}m/T$, where T is a constant that can be interpreted as the average amount of money per agent. The amount of money m that an agent can earn depends on several additional parameters, say $\lambda 1, \lambda 2, \lambda 3, ...$. Since the probability distribution can be normalized as Huang, K. (1987)

$$\int \rho \, d\lambda = 1 \ , \qquad (4)$$

it can be written in general as

$$\rho = \frac{e^{-m/T}}{Q} \ , \quad Q = \int e^{-\frac{m}{T}} d\lambda \ , \qquad (5)$$

where $Q$is known as the partition function.Here λ stays for anyone of the microscopic variables $\lambda_1, \lambda_2, \lambda_3, ...$ or any combination of them. According to the standard approach of statistical thermodynamics, we can introduce the concept of mean value <g> of any function g by means of the relationship

$$< g >= \int g\rho \, d\lambda = \frac{1}{Q} \int g \, e^{-\frac{m}{T}} d\lambda. \qquad (6)$$

In particular, we can calculate now the mean value $< m >= \int m\rho \, d\lambda$and the total differential

$$d < m >= \int (md\varrho + \varrho dm) d\lambda = \int md\rho d\lambda + < dm >. \qquad (7)$$

After some algebraic manipulations it is then possible to prove the equation

$$d < m > = TdS - ydx \qquad (8)$$

$$\text{with } S =< -ln\rho > = \int (-ln\rho)\rho d\lambda \ , \qquad y =< -\frac{\partial m}{\partial x} > = \int \left(-\frac{\partial m}{\partial x}\right) \rho d\lambda \ , \qquad (9)$$

where x is the macroscopic variable associated to λ.Eq.(7) is just the first law of thermodynamics for the mean value of m.This means that once we find a conserved quantity for any economic system, it is straightforward to show that the first law of thermodynamics is satisfied as a consequence of applying the standard formalism of statistical thermodynamics.

Moreover, in statistical thermodynamics it is well known that the partition function can be used to extract all the information about the system under consideration.(Huang, K., 1987). We will therefore calculate Q in two particular cases. Let the conserved quantity m be the function

$$m = c\lambda \ , \qquad (10)$$

where c is a constant. According to Eq.(4), the partition function in this case is given by

$$Q = -\frac{T}{c} e^{-\frac{c\lambda}{T}} \ , \qquad (11)$$

which corresponds to the Boltzmann-Gibbs distribution after the identification

$$N_0 = -\frac{T}{c}, \quad \beta = \frac{c}{T}, \ x = \lambda. \tag{12}$$

In the Colombian income distribution, which is partially described by a Boltzmann-Gibbs density distribution, the variable *x* corresponds to the salary as declared in the GEIH; consequently, according to Eq.(9) the Boltzmann-Gibbs distribution can be seen as following from a simple conserved quantity ($m = cx$) which is proportional to the income declared in each household.

One of the advantages of applying statistical thermodynamics to the analysis of economic systems is that new variables are now available to investigate the system from a different perspective. For instance, the entropy

$$S = 1 + \ln\left(-\frac{T}{c}\right) = 1 + \ln N_0 \tag{13}$$

can be used for the Boltzmann-Gibbs distribution. We see that it is simply a constant which characterizes this segment of the Colombian income distribution.

Consider now the conserved quantity

$$m = c \ln \lambda, \tag{14}$$

for which the partition function can be calculated and yields

$$Q = -\frac{1}{\alpha\,\lambda^{\alpha}}, \quad \alpha = \frac{c}{T} - 1, \tag{15}$$

with the entropy function

$$S = \frac{1+\alpha}{\alpha} + \ln\frac{\lambda}{\alpha}. \tag{16}$$

Eq. (13) corresponds to the Pareto density distribution with the identification

$$N_0 = -\frac{1}{\alpha}, \ x = \lambda. \tag{17}$$

We see again that from the point of view of statistical thermodynamics, the Pareto distribution can be derived from a simple conserved quantity. In this case, the dependence of the function *m* from the income *x* is not linear, as in the case of the Boltzmann-Gibbs distribution, but is given as a logarithmic function. This implies that the properties of the system represented by the Pareto power law function can be completely different from those of an exponential distribution. This can also be seen at the level of the entropy (14) which is now a function of the income *x*.

4. Economics perspective

Certain economic models state that that the incomes of top earners in a given economy form a power law due tothe inherent patters of asset accumulation (Acemoglu, D., Robinson, J. A., 2012).To see this, assume that each individual i in the economy consumes a constant fractionγ of her income,X_{it}, which includes her labor incomes from the pastperiod, Z_{it-1}, and her accumulated assets. Income grows at a stochastic rateof return equal to $r + \epsilon_{it}$, where $E(\epsilon) = 0$. The individual's intertemporal income equation is, therefore, as follows:

$$X_{it+1} = (1 + r + \epsilon_{it})X_{it} - \beta X_{it} + Z_{it} \tag{18}$$

If we normalize by dividing both sides of the equation by that year's GDP (income) per capita *Yt* (which grows at a rate *g*), we get[4]

$$\tilde{x}_{it+1} = \frac{(1+r+\epsilon_{it}-\beta)}{1+g}\tilde{x}_{it} + \tilde{z}_{it} \tag{19}$$

a and dividing by the mean income in the economy, $E[\tilde{x}]$, we have that

$$x_{it+1} = \frac{(1+r+\epsilon_{it}-\beta)}{1+g}x_{it} + z_{it} \tag{20}$$

Where we made use of the fact that this $\tilde{x}_{it+1}$ converges to a stationary distribution, so that $E[x_{it+1}] = E[x_{it}]$. Now, suppose that both z and ϵ are such that we can drop their indice since they do not depend on time, and are the same for all individuals. We then define a complementary cumulative distribution function

$$N(x) \equiv Pr[x_{it+1} \geq x] \tag{21}$$

That is, $N(x)$ represents the proportion of individuals that have a yearly income greater than or equal to *x*. From Eq. (18) we can rewrite this as

$$N(x) = Pr\left[\frac{(1+r+\epsilon_{it}-\beta)}{1+g}x_{it} + z_{it} \geq x\right] \tag{22}$$

$$= E\left[1_{\{\gamma x_{it}+z\geq x\}}\right] \tag{23}$$

$$= E\left[1_{\{x_{it}\geq (x-z)/\gamma\}}\right], \tag{24}$$

[4] In fact, just as x_{it+1} includes the asset rate of growth *r* for the previous period, it must also include the GDP rate of growth *g*.

where 1_p is the indicator function of the event P[5]. Here we have $\gamma \equiv \frac{(1+r+\epsilon-\beta)}{1+g}$.

From this it follows that $N(x)$ is completely determined by x,γ and z. We can rewrite this statement as

$$N(x) = E\left[N\left(\frac{x-z}{\gamma}\right)\right] \quad (25)$$

Now, suppose that $N(x)$ follows a Pareto distribution determined by

$$N(x) = Ax^{-\alpha} \quad (26)$$

when$\alpha \to \infty$[6], we can express Eq.(20) as

$$Ax^{-\alpha} = AE\left[\left(\frac{x-z}{\gamma}\right)^{-\alpha}\right] \quad (27)$$

$$1 = E\left[\left(\frac{x-z}{x\gamma}\right)^{-\alpha}\right] \quad (28)$$

Since $\lim_{x\to\infty} E\left(\frac{x-z}{z}\right)^{-\alpha} = 1$, we can say that Eq. (18) actually conforms to a probability distribution and has a Pareto functional form. Here we have that the lower the value of α, the higher the inequality of income distribution.

Developing further these equations, Acemoglu and Robinson conclude that when the difference $r-g$ gets larger, income inequality rises. Intuitively, the assets of wealthy people give a rate of return greater than the overall growth in product, which gives them an advantage over the rest of the population. In addition, they assert, as β gets lower, income inequality grows, which is explained because a lower β implies a higher marginal propensity to save, which helps rich people maintain an ever-growing (or at least never-decreasing) stock of assets.

The approach of Jones, C., Kim, J. (2014) offers an alternative justification of the regularity of Pareto's distributions, and make use of a simple model of exponential growth of innovation combined with creative destruction. Basically, entrepreneurs create new ideas all the time, and those new ideas help them make money, which in turn increases inequality in the right tail of the distribution. However, every new idea can drive old ideas out of the market, in a creative destruction process, which reduces inequality by restraining the incomes of entrepreneurs (which are, by assumption, part of the high-income segment of the distribution.) According to the model, the interaction between these two phenomena lead to a Pareto distribution of top income. This model is closely related to that outlined in Cantelli, F. P. (1921), in which (as explained in Benhabib, J. (2014)) a variable *H* that determines wealth, like talent, age or ability to implement new ideas, is exponentially distributed:

$$D(H) = pe^{-pH}. \quad (29)$$

It is assumed that income increases exponentially with *H*,

$$x = e^{qH}\ ,\ q \geq 0, \quad (30)$$

Which means that $H = \frac{lnx}{q}$. From this we have that

$$N(x) = N\left(\frac{lnx}{q}\right)\frac{dH}{dx} = \frac{p}{q}x^{-\left(\frac{p}{q}+1\right)} \quad (31)$$

which is Pareto-distributed with exponent $\frac{p}{q}$.

In regard to the other distributions (non-Pareto,) the state of the art is strongly reduced. As for the Boltzmann-Gibbs exponential distribution, used in Angle, J. (1996), a multi-agent model is made in order to simulate the transactions that take place in the economy, under the assumption of a constant amount of money in a given period of time and in the presence of debt (credit.) The economic explanation of such a behavior is, however, largely missing, and is assimilated to the behavior of molecules of gas in interaction. Dragulescu and Yakovenko also warn that the Boltzmann-Gibbs distribution is suitable as a fit for the distribution of money (flows of income) rather than the distribution of wealth (stocks of income and assets.)

5. Conclusions

This work is dedicated to study the income distribution in the case of the Colombian economic system for the years 2002 up to 2012 (data for 2006 and 2007 are not available). We use the official GEIH data employed by the Colombian Government to calculate the Gini index and Lorenz curve. Data come as a sample at the household level, and include the number of individuals in the household and the household's per capita income, as well as an expansion factor that indicates the number of households in the population which are

[5]The indicator function retrieves avalue of one if P is true and of zero otherwise.
[6]This last condition underlines the idea that the power law does not hold for the whole income distribution, but only in the high-income segment.

represented by each one of the households in the sample. To create the large vector of population we expanded the sample by the expansion factor and the number of people in the household. The obtained vector roughly corresponds to the population of Colombia.

The analysis of the Gini Index and the Lorenz curve shows the structural inequality present in the Colombian economy. Moreover, we follow the approach proposed in econophysics for fitting an exponential of the Boltzmann-Gibbs type to the lower tail of the income distribution and a Pareto function to the top income sector. We find that even though the approach holds relatively well in the Colombian case, there is an important deviation in the surroundings of the low-middle class level of income, suggesting that other functions might be more accurate. The main result, however, is that the entire income distribution can be fitted with standard density distribution functions, implying that the Colombian economy can be considered as a thermodynamic system, as stated in econophysics (Quevedo H. Quevedo MN, 2011).

We apply the method of statistical thermodynamics to explain the main income distributions that are present in the Colombian economy, namely the Boltzmann-Gibbs and the Pareto distributions. In both cases, we show that there exists a simple conserved quantity, which is the main input function in statistical thermodynamics and generates the corresponding distribution and the entire thermodynamic information of the system by means of the partition function (Quevedo, H., Quevedo, M. N. ,2016). In the first case, the conserved quantity turns out to be proportional to the income of each household, whereas for the Pareto distribution that quantity is proportional to the logarithm of the income. This difference might be used to understand the structural inequality of the income distribution. Moreover, using the standard procedure of statistical thermodynamics, we calculate the entropy for both distributions and find some differences that may serve to further investigate the properties of the Colombian economic system.

To explain the presence of the Pareto function in the Colombian income distribution, we use certain economic models, stating that the incomes of top earners in a given economy form a power law due to the inherent patters of asset accumulation. These models are widely accepted in the economics community. Unfortunately, non-Pareto distributions have not found so far an explanation in the framework of economics.

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