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Developing Students' Algebraic Thinking in Teaching the Topic of Functions, Graphs, and Its Applications: A Survey on the Perspectives of Math Teachers at High Schools

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Abstract: Algebra is a part of mathematics that plays an important role in solving problems in mathematics and other scientific fields, starting from concepts to algebraic thinking. Therefore, students need to master algebraic thinking skills. To meet that requirement, teachers are the ones who carry out the noble mission of discovering and developing algebraic thinking for students in the most reasonable way. This study was conducted to contribute to the foundation for further studies on the development of algebraic thinking for 10th-grade students. The survey results of 50 high school teachers provide an overview of teachers' perceptions in teaching the topic of functions, graphs and applications and the student's difficulties. According to the research results, most teachers have identified most of the difficulties that students face, but the development of algebraic thinking for students is still limited;having a more comprehensive view of the problem is necessary.
Keywords: Algebraic thinking, teacher's perspective, function

1. Introduction

The concept of a function is central to the high school math curriculum, as it is considered a key tool used in mathematical models describing real-world problems. Today, along with the strong development of technology, meeting the requirements of the knowledge economy requires teachers to grasp innovations in teaching content and methods promptly.

The general education program has been announced by the Ministry of Education and Training, which clearly states the requirements for student's abilities and the characteristics and objectives of the program. This program requires teachers to master the concept of program innovation and equip students with the necessary knowledge and skills in competency-based teaching methods. In addition, teachers must demonstrate their mastery of these innovative perspectives throughout the program.

According to the General Education Program in Mathematics in 2018, the requirements to achieve specific competencies are as follows: "Mathematics contributes to the formation and development of students' mathematical competence (the most concentrated expression of mathematical competence), including the following core components: mathematical thinking and reasoning abilities; mathematical modeling ability; ability to solve mathematical problems; mathematical communication competence; ability to use mathematical tools and means". Good mathematical thinking will lead to problem-solving abilities that can be applied in everyday life and the capacityfor overcoming challenges and focus on the method that delivers the most effective results—considered an important general competence that students must form from the beginning of the learning process [4], [6], [7], [8], [9].

Algebraic thinking is a part of mathematical thinking [5]. There are somealgebraic ways of thinking, such as generalization, abstraction, analytical thinking, dynamic thinking, modeling and organization [3], [13]. Details of the algebraic thinking process can be seen in Table 1.

Table 1: Indicators of algebraic thinking [15]

Algebraic Thinking	*Indicators of Algebraic Thinking*
Generalization	Identify relationships between objects and find the general pattern or shape of a given set of objects.
Abstraction	Use symbols like letters or pictures to represent variables as unknown values based on generalizations.
Dynamic Thinking	Solve problems using various methods.
Modeling	Represent the situation in the problem in a mathematical mode.
Analytical Thinking	Solve problems by using equations to determine the value of avariable as unknown.
Organization	Select and organize data by creating tables, pictures, diagrams,or words describing the problem and therelationship between the overall problem conditions.

Table 1 shows the indicators of algebraic thinking. According to Aprildat and Hakim (2021), two factors affect students' algebraic thinking skills: factors from within students and from outside students [1]. The teacher as an educator becomes an external factor affecting students' mathematical ability. The role of the teacher in choosing how to convey the material greatly influences the effectiveness of the learning process. It can be seen in the research results that students who know how to use learning game applications have a higher level of competence than those who do not.

Based on the algebraic thinking indicators, we realize the difficulty of students in understanding algebraic concepts and the importance of algebraic thinking skills in solving problems in mathematics and other areas. One of the mathematical literatureclosely related to algebra is functions, graphs and applications. In learning this chapter, algebraic thinking is necessary because in interpreting the problem, students must make a mathematical model of the problem within the problem, the transformations, the functional relationships, and the transformations to convert from language to sign the form [10].

Table 2:Requirements for knowledge and skills in teaching the topic of graph functions and applications

Functions, Graphs and Applications	*Requirements to Be Met*
Functions	Recognize real-life models (tables, charts, formulas) leading to the concept of a function. Describe the basic concepts of functions: function definition, set of definitions definition, set of values, covariate function, inverse function, and graph of the function. Describe the geometrical features of graphs of covariate and functional functions inverse. Apply knowledge of functions to solve practical problems (for example, build a first-order function on different intervals to calculate the amount y (payable) according to the number of calling minutes x for a phone plan).
Quadratic function	Set up the table of values of quadratic functions. Draw a parabola is a graph of a quadratic function. Recognize basic properties of parabola such as vertices and opposite axes worthy. Identify and explain the properties of quadratic functions through graph. Apply knowledge of quadratic functions and graphs to solve practical problems (for example, determining the height of a bridge or a gate with a Parabola shape)
Sign of a quadratic triangle	Explain the sign theorem of quadratic trigonometry from observing the graph of the quadratic function. Solve the quadratic inequality. Applying the hidden quadratic inequalities to solve practical problems (for example, determining the maximum height for a car to pass through a tunnel with a parabola shape)
The equation that reduces to a quadratic equation	Solve equations containing roots of the form: $\sqrt{ax^2+bx+c}=\sqrt{dx^2+ex+f}$ $\sqrt{ax^2+bx+c}=dx+e$

The requirements in Table 2 require students to think algebraically about functional relationships, further developing their thinking. Based on the preceding, Table 3 proposes the levels of algebraic thinking in teaching the topic of functions, graphs and applications as follows:

Table3:Levels of algebraic thinking in functions, graphs and applications [11], [12]

Levels	*Manifestations of Algebraic Thinking Ability*
Level 1	Visualization: Know how to write symbols for quadratic functions, quadratic inequalities, and root inequalities; use common and arithmetic language.
Level 2	Using alphanumeric language, hidden symbols bring the problem to algebraic equations.
Level 3	Transformation operations include: Finding definite sets, solving equations, simplifying expressions, and working with expressions and equivalent equations.
Level 4	Find the expression containing the parameter m so that the equation has a solution, no solution.
Level 5	Solve a problem containing parameters.

2. Methods

Qualitative research was conducted to survey teachers' views, the current state of teaching in the direction of developing thinking capacity in general and algebraic thinking in particular, and teachers' perceptions of the difficulties students encounter in learning the above topic.Specific research studies include (1) The level of teachers' interest in teaching in developing general thinking capacity and algebraic thinking; (2) Teacher's comments about the difficulties that students face; (3)Teachers understand measures to develop algebraic thinking and the effectiveness of measures in teaching function topics.The research process is depicted in the following diagram from Figure 1:

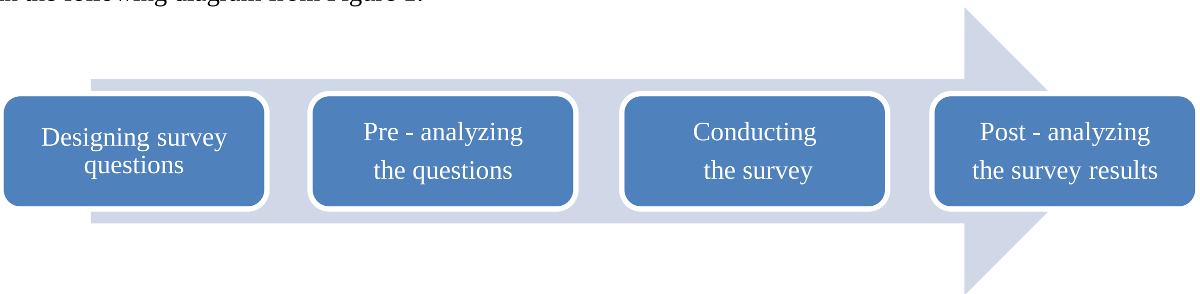

Figure 1: Research process

A survey was conducted with 50 high school math teachers in Vietnam, including high schools in Ca Mau City and Vinh Long province. The survey consists of 8 questions to collect information about teachers' opinions on teaching the topic Functions, graphs and applications to develop mathematical thinking (especially algebraic thinking) and other information related to measures to develop algebraic thinking for 10th-grade students.

3. Results and Discussion

3.1 Question 1: In your opinion, will the development of algebraic thinking for 10th graders contribute to developing students' competencies and qualities per the views prescribed in the General Education Program in Mathematics 2018?

Table 4: Statistics of teachers' opinions on Question 1

Scale	*Strongly Disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly Agree*
f	0	0	4	30	16
%	0	0	8	60	32

The General Education Program in Mathematics 2018 clearly states the requirements for specific competencies, whereby Mathematics contributes to the formation and development of students' mathematical competence (the most concentrated expression of computational competence). One of the core components is the ability to think and reason mathematically. Algebraic thinking is a part of mathematical thinking; the development of algebraic thinking contributes to the development of thinking and reasoning capacity in particular and students' general capacity and qualities. According to Table 4, the results show that the rate of consensus from teachers in some high schools is 92%. Due to improvement and development; this encourages the study of the subject in a new direction.

3.2 Question 2: Teachers indicate their level of interest in the following concepts:

1 Mathematical thinking
2 Abstract thinking
3 Creative thinking
4 Critical thinking
5 Algebraic thinking

Table5: Statistics of teachers' opinions onQuestion 2, including ideas in order from 1 to 5, respectively

Scale / *Items*	*Strongly Disagree*		*Disagree*		*Neutral*		*Agree*		*Strongly Agree*	
	f	%		%	f	%	f	%	f	%
2.1	0	0	3	6	7	14	40	80	0	0
2.2	3	6	0	0	14	28	30	60	3	6
2.3	3	6	0	0	3	6	32	64	9	18
2.4	0	0	6	12	6	12	29	58	9	18
2.5	0	0	15	30	25	50	10	20	0	0

Today, with the development of information technology, to meet the requirements of the 2018 General Education Program, teachers must learn by themselves, be creative, and promote the role of teachers, which is to change strongly from the position of being a teacher. "teacher" to the position of "organizing, checking, and directing" students' learning activities. Implement more effectively and thoroughly the requirements of the "learning by doing" teaching method. The survey in Table 5 shows that most of the surveyed teachers promote their self-study and self–improvement roles by understanding the students' thinking; an average of 65% of teachers are regularly and very often interested. Mathematical thinking comprised 80%, abstract thinking 66%, creative thinking 82%, and critical thinking 76% of all types of thought. Algebraic thinking is interested in only 20% of teachers, although research on algebraic thinking has been mentioned a lot because of Mathematics, including algebra. It plays an important and indispensable role in education in general and in developing students' thinking from preschool to high school. If algebraic thinking measures are implemented well, they will help students and even teachers perform well in their teaching and learning tasks.

3.3 Question 3: In your opinion, how is the development of algebraic thinking for students assessed?

Table 6: Statistics of teachers' opinions on Question 3

Scale	*Very unnecessary*	*Unnecessary*	*Neutral*	*unnecessary*	*Very unnecessary*
f	0	0	12	28	10
%	0	0	24	56	20

Question 3 was designed with the purpose of testing teachers' interest in developing algebraic thinking for students. Results obtained from Table 6, the number of teachers who realize the importance of developing math problem-solving capacity for students accounts for more than half of the total number of teachers participating in the survey (38 teachers, accounting for 76%). With such a result, we can see that the research on applying measures to develop algebraic thinking for students will be easily accessible to most high school teachers.

3.4 Question 4: What is your opinion about the obstacles for students when learning functions, graphs and applications through the following expressions?

Table7: Statistics of teachers' opinions on Question 4 (Only list the cases of agreeing, completely agreeing)

Obstacles for Students	*f*	*%*
Students believe that the equal sign represents one-way math that processes the right output from the left input	30	60
Focus on finding specific answers	37	74
Do not recognize commutative and distributive properties	23	46
Do not use mathematical symbols that represent relationships between quantities	23	46
Do not understand the use of letters as a general number or as a variable	23	46
Having great difficulty performing math on unknowns	30	60
Having great difficulty performing math on the unknowns Not understanding that equivalent transformations on both sides of an equation do not change its equation.	30	60

Question 4 was designed to discover the difficulties that students often encounter from the teacher's perspective in the teaching process, especially in developing students' algebraic thinking. The results obtained from Table 7 show that the most serious difficulty for students is that 74% of the teachers agree that they still carry heavy arithmetic thinking and calculate with direct numbers that they have done. In elementary and middle school, years of practice are needed to organize activities to develop students' algebraic thinking. Next is the concept of the "=" sign; the unknowns and the transformations are also quite a big obstacle for students (with 30 teachers observing, a rate of 60%). They were followed by difficulties related to symbols, manipulation of unknowns, and properties of expressions (accounting for 46%). Most teachers who participated in the survey have teaching experience and have learned about students' difficulties. Using measures to develop algebraic thinking in teaching to develop mathematical thinking and reasoning abilities for students has become necessary, with measures to overcome and limit students' difficulties.

3.5 Question 5: According to teachers, to develop algebraic thinking for students, what strategies can be applied in the teaching process? (You can choose more than one strategy)

Table8: Statistics of teachers' opinions on Question 5

Strategies	*f*	%
Engage→Explore→Explain→Elaborate→ Evaluate (5E learning model)	40	80
Use technology (games, spreadsheets, software, etc.)	23	46
Use manipulations	10	20
Use multiple representations	13	26

For STEM education programs, the 5E model becomes an effective tool to help learners and teachers feel that they receive lessons systematically and seamlessly and can develop according to their needs, psychology of self-discovery and knowledge construction. Many recent studies have shown that the 5E model brings many positive effects on teaching work, which is reflected in the subject characteristics in the General Education Program in Mathematics, which is institutionalized in the content in textbooks, increasingly popular with teachers. Therefore, 80% of teachers appreciate this content in developing students' algebraic thinking. According to most respondents, the application of active teaching methods in teaching is more agreed upon by teachers. However, in Table 8, all of the above strategies develop algebraic thinking for students, which should be paid attention to when imparting knowledge to obtain the highest efficiency.

3.6 Question 6: According to you, what content in the 10th-grade math textbook can be exploited to develop algebraic thinking for students? (You can choose more than one option)

Table 9: Statistics of teachers' opinions on Question 6

Content	*f*	%
Propositions and Sets	27	54
Inequalities and systems of first-order inequalities with two unknowns	33	66
Functions, graphs and applications	30	60
Combinatoric algebra	20	40
In-plane coordinate method	13	26

The question was raised to test the consensus of teachers when choosing the most expensive situations to build measures to develop algebraic thinking for students. The survey results in Table 9 are not unexpected. Functions and equations are the two areas of most interest. In particular, functions are central in the high school math curriculum, relating to many other concepts about equations, inequalities, and mathematical models that describe practical problems.

3.7 Question 7: To discover and develop algebraic thinking for students at each level, teachers should choose the following contents and arrange them from low to high level.

1. Perform arithmetic operations with extended objects, such as counting and math operations.
2. Show object generalization.
3. Algebraic generalization activities: Forming expressions and equations. Show a generalization of the problem.
4. Consider the relationship between variables.

5. Use parameters and variables in generalization operations. Show symbols.
6. Parameter handling.
7. Analyze algebraic structures.

This question was asked to re-evaluate teachers' interest in developing algebraic thinking. Most choices are correct in the first three steps, the common level of student thinking seen by the teacher. Completing the research model, only seven teachers (14%) gave the correct answer; this will be a suggestion to promote the implementation of full steps to help teachers and students achieve the highest results in their teaching and learning process. Through relatively well-equipped teachers, teachers can assess students' algebraic thinking levelsand find appropriate measures to develop this thinking for them to go further.

3.8 Question 8: In which teaching situations do you think the development of algebraic thinking will achieve high results? (You can choose more than one answer).

Table10: Statistics of teachers' opinions on Question 8

Situations	*f*	*%*
Teaching concepts	13	26
Teaching rules and formulas	30	60
Teaching, solving exercises, practicing	30	60
Teaching experience	27	54

The question was designed to investigate teachers' conceptions of building situations to develop algebraic thinking for students. The results obtained from Table 10 show that most teachers find it most feasible to design situations in teaching with problem-solving, practice, formulas, and rules (rate of 60%), experiential teaching situations (rate of 54%), and teaching of concepts (rate of 26%). The key to students' algebraic thinking lies in choosing solutions from relevant mathematical knowledge, so students need to understand concepts and theorems to solve problems well.Measures to develop algebraic thinking will help students better understand math concepts by building activities that promote students' thinking about a math concept.

4. Conclusion

The survey results show high school teachers' level of understanding about their ability to think and reason mathematically and algebraic thinking. Another surprising result obtained from this study is that almost all of the teachers participating in the survey believe that the problem of algebraic thinking is new to teachers because it is largely understood to be the default in mathematical thinking, which is also reasonable according to statistics on search terms algebraic thinking based on databases most of the research works on algebraic thinking related to elementary school students, middle school, referring to a small number of high school students. The survey results show that teachers' judgments about the difficulties that students often face in acquiring knowledge create cognitive obstacles that negatively affect the teaching and learning of algebra.

Regardingfunctions, graphs and applications, the survey results also show that teachers pay special attention to constructing teaching situations in teaching and solving exercises, exercises, formulas, and rulesis the most feasible; this can also be considered as an advantage for the application of measures to develop algebraic thinking in teaching practice.

In addition to the above results, the study has some limitations that must be considered. First, the research results are still local because only 50 teachers from neighboring provinces participated in the survey. As a result, findings only represent teachers' perceptions of students as individuals in this regard. Therefore, it can be said that the findings of this study are local.

5. Recommendations

Professional training is a regular and continuous job for those working in education. Therefore, it is necessary to equip teachers with teaching strategies to develop thinking capacity, including algebraic thinking [2]. New studies with larger samples can be conducted, expanding the survey geographically or by grade level [14]. It is feasible to survey the state of research and application of algebraic thinking measures in teaching mathematics to future school teachers, more importantly conceptually and practically. Learning and applying measures to develop algebraic thinking in teaching functions, graphs and applications is possible and will certainly contribute to developing and improving students' thinking. It would be wise to take this into account as a potential substitute.

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