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# Comparison of existing opinions on Volume in Plato's "Menon" with modern-day studentsin Greece

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**Abstract:** In this paper an attempt is made to repeat historical events related to how modern students perceive changes in the dimensions of objects (volume) in relation to those described in ancient texts. The research was undertaken by us in the 1st Lyceum (senior high school) in the academic year 2011 in a large island city. There were 432 children in the sample. It would seem that modern students face the same problems in explaining how the volume of an object changes when changing its dimensions of width, length and height.

## Introduction

The purpose of this work is to examine the ability of the studentsto predict changes in volume when two or three dimensions of a solid object are changed. This will become similar to the famous example of the slave in Plato's dialogue *Meno*, who was asked todraw a square having two times the area of a given square and first proposedto double the side of that square (see, e.g., Berté, 1992, 1993, Daumas, 1989,Bork,Dorren, Jansssens,Verschaffel, 2007). Or it recalls the legendof the Athenians doubling allsides of their altar at the request of the oracle of Apollo (Smith, 1923).

The research question is, therefore, whether the students can predict changes in volume when there are changes in 2 or 3 dimensions of an object.

## Theoretical framework

In the theoretical framework we will concentrate on research that has to do with the role of the illusion of proportion when there is a change of volume when changing the dimensions of the length, width or height of one, two or three of them.

In the past, proportional reasoning was a good mathematical tool with which we could tackle problems in physics, chemistry, biology technology and many other sciences (Freudenthal, 1973). By looking at the bibliography we observe that there are many descriptions which attempt to define the concept of proportional reasoning. According to Inhelder& Piaget (1958) the basic characteristic of proportional reasoning is that it focuses on the description, forecast and evaluation of the relationship between two other concepts, that is to say it includes a second degree relationship and not only a single relationship between two discrete objects or concepts. In essence this means that proportional reasoning can focus on a second degree relationship which includes an equal relationship between two reasons (Lamon, 1999, Leinhardt, Zaslavsky, and Stein (1990), several studies showing that students of different ages have a strong tendency to produce a linear pattern through the origin when asked to graph non-linear relations, for example the growth of the height of a person from birth to the age of 30.

Another well known example where the linear relation as a phenomenal tool of description fails (Freudenthal, 1983) is the case of the non-linear behaviour of area and volume under linear multiplication. Students in fact fail to see the non-linear character of the increase or decrease and handle the relations between length and area or between length and volume as linear instead of quadratic and cubic, respectively (Modestou&Gagatsis, 2007). Students' former real life practices with enlarging and reducing operations do not necessarily make them aware of the different growth rates of lengths, areas and volumes (De Bock, Verschaffel, & Janssens, 2002). Consequently, they apply the linear scale factor instead of its square or cube to determine the area or volume of an enlarged or reduced figure. This should not be overlooked. As Freudenthal (1983) points out understanding that multiplication of lengths by d, of areas by $d^2$ and of volumes by $d^3$ is highly associated with the geometrical multiplication by d is mathematically so fundamental, that phenomenologically and didactically it should be put first and foremost. Students should be able to distinguish that area and volume are proportional to length only when the variable(s) related to them are held constant.

In the *NCTM Standards* that "... most students in Grades 5–8 incorrectly believe that if the sides of a figure are doubled to produce a similar figure, the area and volume also will be doubled" (National Council of Teachers of Mathematics, 1989, pp. 114–115). In other words, students strongly tend to see the relations between length and area or between length and volume as linear instead of, respectively, quadratic and cubic, and, consequently, they apply the linear scale factor instead of its square or cube to determine the area or volume of an enlarged or reduced figure.(book de Docck)

If teachers knew more about the growth of students' conceptual understanding of length, area, and volume measurement, they would be better able to teach these topics. And if they better understood how students relate measurement in the three domains, they would be better able to relate the teaching of each attribute to students' understanding of the others.

A proportion students refers to the equality of two ratios $a/b = c/d$ (and thus to a relation between four measurements), such as: "In one urn there are 20 white balls and 40black balls. The other urn has 50 white and 100 black balls. The chance fordrawing a white ball is equal for both urns since 20/40 = 50/100." The concept of linearity or proportionality (or a linear or proportional *relation* between quantities) refers to the equality of a multitude of equal ratios: $a/b = c/d = e/f = \dots$

Second, there is a vast amount of theoretical and empirical research on how andwhy drawings and diagrams may enhance people's ability to represent and solve(mathematical) problems. In this respect, we refer the reader to Larkin and Simon's(1987) famous article 'Why a diagram is (sometimes) worth ten thousand words,' butalso to several more recent meta-analyses on the role of diagrammatic and pictorialre presentations in (mathematical) problem solving (Aprea & Ebner, 1999, Hall, Bailey,& Tillman, 1997, Reed, 1999,VlahovicStetic, 1999) as well as to the literature on the role of heuristics in skilled (mathematical) problem solving in general (Collins, Brown, & Newman, 1989,De Corte et al, 1996,Polya, 1945). According to all of these authors, the heuristic of visualising the problem using a drawing or diagram does not, of course, guarantee that one will find the solution of a given problem. But because it induces a systematic analysis and elaboration of the problem situation, because it enhances a planned solution of the task,and because it can also be used in interpreting and checking one's answer, makinga drawing or a diagram, it is generally considered a very helpful and successful cognitive tool in (mathematical) problem solving. In accordance with the active, constructive, and self-regulated view of effective learning (De Corte et al, 1996), the positive effect of visualisations is considered to be even stronger when they are made by the learners themselves rather than merely being given to them by the teacher or there searcher, since self-made visualisations stimulate an even more deep-level and mindful approach to the task (Aprea & Ebner, 1999,Dirkes, 1991). Of course, although generating a drawing increases the chance that a problem will be conceptualised correctly, a drawing that reflects an incorrect understanding of the problem will be of little help for the problem's solution (Van Essen & Hamaker, 1990, De Bock et al., 2003)

With these elements as a base, the students have a tendency systematically to use the proportional model even in situations for which it is not suitable. It appears to be connected with the absence of the skill of recognising and determining the problem and hence there is late proportional awareness on the part of the students (Modestou&Gagatsis, 2006, 2007).

Furthermore it means that their understanding of proportionality lacks some structural links with significant adjoining geometrical questions. So again there is a tendency systematically to use the proportional model even in situations for which it is not suitable. It appears to be connected with the absence of the skill of recognizing and determining the problem.As a suggested remedy, it is proposed to displace the emphasis from computing correct numerical answers to building appropriate mathematical models (see Van Dooren, De Bock, Depaepe, Janssens, &Verschaffel, 2003).

Leinhardt and his colleagues dealt with the classification of misinterpretations in the thinking of students with regard to graphic representations(Leinhardt et al 1990) .

De Bock and his colleagues also investigated the above phenomenon, effecting a series of empirical studies to demonstrate the deeply-rooted tendency of students aged 12 – 16 to perceive non-proportional phenomena as proportional (De Bock et al, 1998, 2002b, το 2003).

According to Hughes-Hallett, (Hughes-Hallett et al 1996), a significant element in the correction of the illusions that accompany the student, is that we view a concept from a triple perspective. He maintains specifically that it is good that certain subjects be taught graphically, numerically and analytically. The aim of this is to give the student three equally balanced sides so that he/she sees the same concept from various perspectives.

These errors take the form of epistemological obstacles since they are not related to the individual who made them but are both ontogenetic and instructive in origin (Radford, Boero& Vasco 2000). They are characterized by their re-appearance in all epochs in which mathematics have been used and have to do with the way in which the individual handles the nature of mathematical problems (Freudenthal 1983). As Brousseau notes (Brousseau 1997), those obstacles which may be characterized as epistemological are those which cannot and must not be avoided, exactly because they play a decisive role in learning. So the obstacles hold out against the occasional appearance of contradiction, we situations and the improvement of the student's knowledge (Brousseau, 1997).

The experiences even of today's students with the magnification and shrinking of shapes, do not lead them to realize the different rates of increase in length, area and volume (De Bocketal,2002). As a consequence students tend to treat the relationship between length and area as well as length and volume as linear instead of square and cubic respectively.

There is a vast amount of theoretical and empirical research on how and why drawings and diagrams may enhance people's ability to represent and solve mathematical problems (De Bock,2003 from where you took this). There are famous articles on the role of diagrammatic and pictorial representations in (mathematical) problem solving (Aprea & Ebner, 1999) In accordance with the active, constructive, and self-regulated view of effective learning (De Corte, et al, 1996), the positive effect of visualisations is considered to be even stronger when they are made by the learners themselves rather than merely being given to them by the teacher or the researcher, since self-made visualisations stimulate an even more deep-level and mindful approach to the task (Aprea & Ebner, 1999, Dirkes, 1991). A drawing that reflects an incorrect understanding of the problem will be of little help in the problem's solution (Van Essen & Hamaker, 1990, De Bock, 2003).

According to all of these authors, the heuristic of visualising the problem using a drawing or diagram does, of course, not guarantee that one will find the solution of a given problem. But because it induces a systematic analysis and elaboration of the problem situation, because it enhances a planned solution of the task, and because it can also be used in interpreting and checking one's answer, makinga drawing or a diagram is generally considered a very helpful and successful cognitive tool in (mathematical) problem solving. In accordance with the active, constructive, and self-regulated view of effective learning (De Corte et all, 1996), the positive effect of visualisations is considered to be even stronger when they are made by the learners themselves rather than merely being given to them by the teacher or there searcher, since self-made visualisations stimulate an even more deep-level and mindful approach of the task (Aprea & Ebner, 1999, Dirkes, 1991). Of course, although generating a drawing increases the chance that a problem will be conceptualised correctly, a drawing that reflects an incorrect understanding of the problem will be of little help for the problem solution (Van Essen & Hamaker, 1990).

Regardless of the type of figure or object (square, circle, sphere, irregular figure, etc.), an enlargement/reduction[2] times 2 of factor K means an enlargement/reduction of lengths and perimeters of factor K and a corresponding enlargement or reduction of surface areas to factor $K^2$ and volumes to factor $K^3$. In particular, these studies (Modestou et al., 2008, Modestou & Gagatsis, 2006) tried to put in question the application of the linear model in area and volume problem solving. On the contrary this study is motivated by theoretical concerns which come from the lack of a comprehensive theoretical framework on spatially organized quantities capable of supporting different kinds of reasoning and problem solving, including the pseudo-proportional problems. In fact, there are not many studies, which focus on the development of models explaining the structural relationships between different types of problem solving on area and volume. This study intends to give insight into three different dimensions of students' abilities in solving geometrical problems on area and volume, including usual computation problems, pseudo-proportional problems and unusual ones, through the proposition of a coherent structural model. Identifying the structural organization of these types of geometry problem solving abilities may further illuminate the phenomenon of the illusion of linearity based on the conjunction of students' way of handling the pseudo-proportional problems and the problems of different reasoning requirements on the same subject.

For example, in a teaching experiment with 11$^{th}$graders, Van Deyck [19] offered students a scatter plot in which the pattern of the dots showed a parabola. Although these students already had encountered several non-linear models in their mathematics curriculum (including quadratic models), they typically argued that there was no relation between the two represented variables, and they saw a further argument for the absence of a relationship in the Pearson correlation coefficient in the corresponding data set being nearly zero. Apparently, when looking at the graph, the students only searched for a straight-line pattern, and they were moreover not aware that a Pearson correlation coefficient can indicate the presence of linear relations only. This is something very significant. Although, from a mathematical point of view, linearity and straight-line graphs are closely related, it remains unclear whether students always make this link, and really conceive the relation that they represent as a linear one, because if so, something like that would be catastrophic (they would burn out the sensor) since it is not possible for the students, with the elements that we have given them to envisage the characteristic function of the sensor.

## Participants – Methodology

The historical event that refers to a change in volume becomes an attempt (below) to verify it. Specifically, we carried out a survey in the 1$^{st}$ Year of Lyceum in a large island city in the school year 2011. 4 schools took part including an Evening High School. The sample was made up of 432 pupils. Approximately

56% of the participants were girls. The questions were given in writing to the students after the researcher read them aloud in the presence of their class teacher. The students then gave their answers in writing.

## Change of area - volume

Undoubtedly, the most famous and most often quoted example of the occurrence of the linearity illusion can be found in Plato's dialogue *Meno* (Berte´, 1993, Rouche,1992) in which a slave, when asked to draw a square having two times the area ofa given square, initially proposes to *double the side* of that square. So, the slaves pontaneously applies the idea of linear proportionality (between length and area) and changes his mind only when Socrates helps him in diagnosing and correcting the error in his reasoning by confronting him with a drawing. Recent research by De Bock et al. (1998, 2002), that will be summarised in the next section, provides strong evidence that—more than two millennia later—an alarming number of students still consider the relations between length and area or between length and volume in similar geometrical figures as linear instead of quadratic or cubic, andapply the linear scale factor instead of its square or cube to determine the area orvolume of an enlarged or reduced figure (Bock,Verschaffel, Janssens, Dooren, Claes,2003)

It may also be mentioned that in 430BC in Delos the Athenians asked the Oracle of the god Apollo how they should deal with the plague that had turned their city into a desert, Then Apollo "indicated" to them that in order to stop the disease they would have to double the size of his altar. Even though they doubled the side of the altar the plague was not eliminated. (Smith, 1923 from De Bocκet al. 2007).

**The problem given to the students was:**

In answer to a question put by the Athenians an oracle answered that they could deal with the plague that had ravaged their city if they doubled the size of the altar of the god to whom the city was dedicated. The Athenians doubled each side of the altar. Let us suppose that if they had given what the god had asked of them the plague would have stopped, was what the god had asked of then given by the Athenians

In order for the reader to have a picture of the answers given by the students, a few of them are presented below (figures 1,2,3).

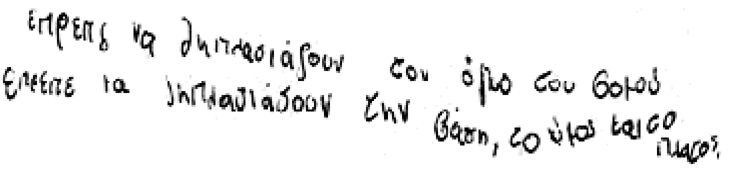


**Figure 1**

In Figure 1 the student answers that doubling the volume should be accompanied by the doubling of each side (i.e. length height and width of the cube). It is noted that the majority of the students (78%) had difficulty in answering the specific question and took refuge in solving the problem using erroneous algorithms. For example the student in Figure 2, uses the formula $(2 \cdot 2)+2$, which is wrong.

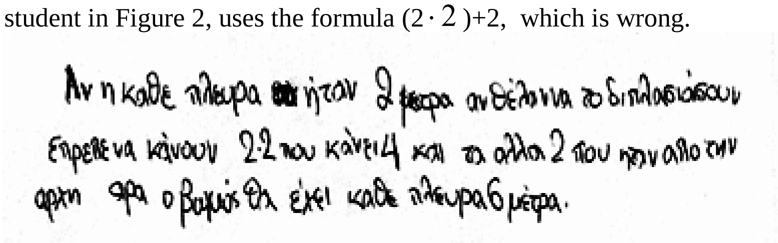


**Figure 2**

The students' difficulties accord with the research of the US International Council of Teachers which mentions that most students from Primary 5 to the 3rd year of Junior High School consider that doubling the side of a shape also doubles its area and volume (National Council of Teachers of Mathematics 1989, reference from De Bocketal, 1998).

Some students however predict correctly, realising that the volume increases more than the area. One such answer can be seen in figure 3.

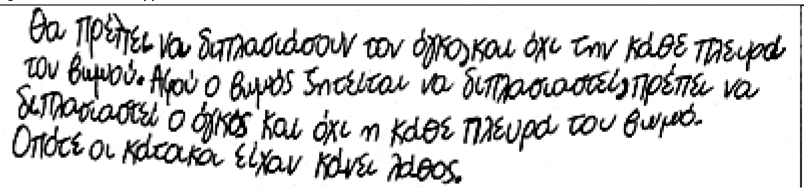



**Figure 3**

Only 4% of the students said that after the edge of a cube is multiplied by factor a, the surface multiplies by $a^2$. ($E_2=a^2 \cdot E_1$) and so the volume by $a^3$. This answer was given by more than 80%. These students consider that there will be a doubling of volume since there is a doubling of each side of the parallelogram.

**Table 10**

| Grouping of students' answers | Number of students | % |
|---|---|---|
| Doubling each side of the altar doubles the altar | 277 | 83% |
| Non-proportional– Correct answer | 13 | 4% |
| Don't know –No answer gicven | 42 | 12% |

What is most noteworthy, however, is that despite the fact that explanations and other examples were given by the teacher, the students, in other similar problems in the near future, (1 month later)could not avoid "erroneous linear solutions." Thus statistically there is no significant difference in the students' answers (sincep=0,95>0,05) in the answers give a month later compared to those already described above. This reinforces the view already mentioned above that their intuitions "resist."

### 1.2. Attempting to correct their reasoning. The superiority of the computer

Next the same solid bodies with their changing dimensions were investigated in different ways so as to obtain different volumes.

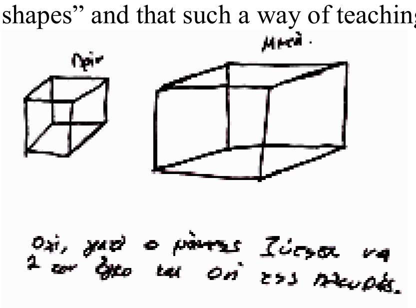
These ways were to represent the (above) objects:

A) On paper (figure 4),

B) In a table,

Γ) **On the computer** having first measured them with a ruler in order to insert them into the corresponding scales.

In our case we observed that there was a greater "correction" of the students' senses when using a computer, more so than the students actually touching the various solids, measuring them or drawing them on paper. In particular, when in some way a cube or cylinder or any other solid shape was entered into the computer, the student could follow the changes in volume (dimensions) on the screen, understanding these changes better (Avgerinos& Marinos 2009). The above conclusions are in line with Freudenthal (1983) who states that enlarging or shrinking shapes is an important action for a student to understand "changes in solid shapes" and that such a way of teaching requires greater emphasis and more time.

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### 1.3 Conclusions

The students who took part in this research were students of Evening High School who work in the morning hours. They are more concerned with realistic mathematics than students in other types of school.

The majority of the researchers that have worked on students' reasoning in problem solving concerning area and volume and specifically on the tendency to deal linearly with non-proportional tasks agree that the obstacle of linearity isvery difficult for the students to overcome (De Bock et al., 2002, Modestou & Gagatsis, 2007, Van Dooren et al., 2004). This is related to the fact that the obstacle of linearity is not a didactical obstacle or a developmental one but anepistemological obstacle (Modestou & Gagatsis, 2007).

The results of such attempts may help teachers at the high school levels to place emphasis on certain dimensions of geometrical problem solving and use more appropriate pedagogical approaches to teaching them. By these means students can be assisted in constructing a solid and deep understanding of length, area and volume and their different growth rates, in interpreting different types of problems on these notions appropriately and in employing the solution processes that correspond to them

The phenomenon of the "misuse of proportion" that occurs in our research is not due to any absence of knowledge. This is also shown in our results which indicate once again that the errors resulting from the application of the linear model in non-proportional situations are not temporary, transient, or random but are deeply rooted in the students' thinking. This was also shown in the re-examination shown above. The phenomenon of the illusion of proportion is not a common mistake and therefore cannot be easily corrected (Kontogiannopoulos, 2010), since the student's resort to it as a safe strategy for solving problems and, as shown in our research and mentioned above, the students resort to it without any doubts. (De Bock et al., 2002). Errors may be repeated and resist any supporting means aimed at tackling the problem (De Bocketal., 2003), as also appears in the above research. So despite being given a sheet of papers, this didn't help. At the same time the view is reinforced that the most likely way to deal with the epistemological barrier of proportion is to organise a teaching situation (Modestou & Gagatsis, 2007, 2010) which forces the students to challenge and ultimately reject on their own the linear model which is only suitable for problems that meet certain criteria.

Students' thinking leads to a mistaken conviction that the analogue model has a "universal" applicability and that any numeric relationship can be seen as linear (Freudenthal, 1983). Furthermore it can be mentioned that the above "weakness" is linked to the absence of the "skill" of identifying and defining a "problematic situation." Thus there is a tendency for students to systematically use the analogue model in situations for which it is not appropriate. Students are not able to analyse whether or not there is an analogue relationship between the different quantities of a problem and this is a very important cause of the illusion of proportion (Modestou & Gagatsis 2010).It was further observed that the student's intuitive perception and understanding of reason and proportion affect his/her performance. This is consistent (apart from our own research) with surveys conducted by Noss, R., & Hoyles, C. (1996)

As written in the theoretical context, many students consider that if one of the dimensions of a solid is doubled then its volume will also double or be subdivided as the Athenians believed in the Delos problem. (Modestou & Gagatsis, 2007Modestou,Elia, Gagatsis, & Spanoudis, 2008, Outhred & Mitchelmore, 2000). We ourselves were also led to similar results, and as mentioned above and as argued by Smith, 1923 and by De Bocкet al. 2007, the fact of the erroneous evaluation of the increase in volume of the altar is an event that has existed for a long time (as in our own students). Lo (Lo,1993) also considers that the concept of proportion is not something that the students have or lack, and that the difficulties in understanding proportion may be due to an inappropriate method of teaching in the classroom.

The results of such attempts may help teachers at the high school levels to place emphasis on certain dimensions of geometrical problem solving and use more appropriate pedagogical approaches to teaching them. By these means students can be assisted in constructing a deep, solid understanding of length, area and volume and their different growth rates, in interpreting different types of problems on these notions appropriately and in employing the solution processes that correspond to them

The students' proportional thinking is largely related to the ability to recognise structural similarity or contribution, and is the result of individual adaptation and reconstruction of existing knowledge. The use of the computer can help the student to realize the changes that occur in space in a very easy way after seeing the changes of his/her computer. The present is the subject of another investigation

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