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The Explanations written by students in their exercise books regarding diagrams of the relationships of weight and volume of bodies immersed in water

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Abstract: Discussion of the pupils' way of thinking and their mistakes in concepts related to the maintenance of or change in volume "discontinuous quantities (of liquids)" takes place with the written explanations given by the pupils considering that they have a container which contains water up to a certain height.
It is observed that the pupils are confused and think that we have a change in volume if we use different materials even when we tell them that we have the same volume. They also find it difficult to recognize that two materials of the same weight but of different substance will occupy the same volume. The students carried out the experiment and wrote it up on worksheets, considering that the bodies are immersed in containers of water.
Keywords: Jean Piaget, developmental stages, volume, representations

1. Introduction

The theory of the Swiss biologist and psychologist JeanPiaget (1896-1980) [1] who studied the cognitive development of children, supports the idea that the developing child builds cognitivestructures, in other words mental maps - the forms by which he understands and reacts to natural experiences in his environment. Piaget proved that the cognitive structure of the child increases in complexity as the child grows [2]. He points out four (4) developmental stages A) **Sensorimotor** (from birth to 2 years old): through a natural interaction with his environment the child constructs a series of concepts about reality. This is the stage in which a child does not know that physical objects continue to be present even when he does not see them (objectpermanence). B) The **Preoperational - Pre-reasoningStage** (from 2 to 7 years): the child cannot yet form abstract concepts and requires specific physical situations. C) The **Concrete OperationsStage** (from 7 to 11 years): as physical experience is accumulated the child begins to form concepts and to solve abstract problems e.g. numerical equations with numbers rather than objects D) the **Formal OperationsStage** or **Abstract Thought** (from 11 to 15 years): at this point the cognitive structures of the child resemble those of an adult and include abstract logic. The variety and brilliance of the experiments which Piaget devised allowed him to reveal the main characteristics of children's thinking. It is maintained in these studies that pupils from an early age develop clear, egocentric behaviour. The pupils cannot keep two dimensions in their minds at the same time, but this element recedes as the child grows. In the following table we have two balls of plasticene which are converted (Boston, Deliege 1998), [3]

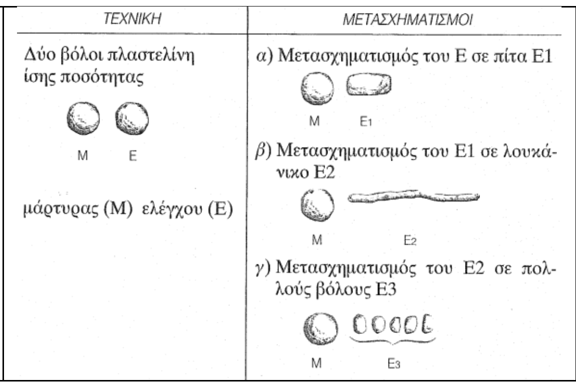

Figure 1: Transformations of plasticine

Table 1: Observed behaviour in the maintenance of continuous quantities (the volume & weight of matter).

Age	Observed behaviour in the maintenance of continuous quantities (the volume & weight of matter).	Observed behavior in the maintenance of continuous quantities (of liquids)
3-4 years	The subjects give answers as follows: E1 is much more because it is longer. E2 is much more because it is thinner E3 is much more because it contains a lot of balls. Therefore complete conformity of thought is observed on what happens with each form that the plasticene takes.	At this age the child support his reasoning exclusively and solely with the height of the liquid reaches. He thinks that E1 has less than E2, and that E2 has much more than E. However when we remove the small ball from the small containers the pupil believes that the E3 (containers) are more polychromatic than M.
5-6 years	At this age the idea of the maintenance of matter appears, but it is something that can easily change. Thus there is agreement that M=E1, but it is disputed that M=E2.	If we ask the child to place in E1 (i.e. the container which is empty) as much water as there is in M, he puts in as much as will rise to the same height as in M. Then the pupil has doubts and takes out a little. He notices a difference in width. But he cannot at the same time pay attention to both height and width. (Oscillations)
7-8 years	At this age children maintain the quantity of the matter regardless of any transformations it acquires. That is to say the children are able to justify why the quantity of plasticene remains the same.	The child takes several ten years back out of the height and the width of the container. He also understands that these dimensions can be altered. Specifically As the height increases and the width decreases the liquid in the two containers is the same and contrary.
9-10 years	The concept that the weight of the volume is less than the matter. From this it is realized that we must wait a few years more before there is certainty in the child that the weight of the object does not change when the shape of it changes.	
11-12 years	We have to wait four years (from the maintenance of matter) until the child accepts that the volume of an object does not change when its shape changes.	

During the entire developmental stage the child experiences in his environment using his already existing mental maps. If an experience is repeated it suits and is absorbed (assimilation) easily into the cognitive structure of the child, soas to maintain his mental equilibrium. If the experience is different or new, the child loses his mental equilibrium and changes his cognitive structures to accommodate these new conditions. In this way the child acquires many cognitive structures one by one.

The teacher, according to developmental theory, must design the material of the lessons to suit the developmental stage of the students. He must also place emphasis on the decisive role played in the student's learning by the child's experiences or his interaction with the environment. This intervention can also take place within a standardized framework, Depending on how the students as a group understand concepts of weight and volume or how they deal with problems in their everyday life which contain a combination of the above concepts (Campione et. al,1984)[4]. This can take place with the help of diagrams. A diagram cannot be said to have the same possibilities of representation as language. A particular material symbol and a even be attributed to an intellectual symbol or concept (Kaput, 1987) [5](signifierandreferentmaterialsign) and another to another symbol or concept. Thus with diagrams we can represent the entirety of relationships which exist between the individual elements which compose an object or a situation. On the other hand diagrams and proportional representation in general depict only situations, setups and results of actions without being able to represent actions or transformations where a field which has the attributes of the language of physics or algebra is needed.

By the term "translation of representations" we mean the psychological process which takes place when we are transported from one system of representation to another. Lesh (1979)[6] stresses the role of translations in the solving of problems. In order for the translation to take place at least two forms of representation are needed. The source, i.e. the initial representation, must be given from the optical angle of the second. However, the optical angle, the way in which a student in "sees" the representation, is the result of teaching, the emphasis of which is not on the student copying what the teacher does but on the successful organizations of his own experience [7].Translation between different representations of the same object can be considered a necessary prerequisite for the solution of mathematical problems [8].

2. Direction of the Research.

The students must record on paper how they perceive the case where a substance is introduced into a container full of water under the following circumstances:

A. If, in the student's thinking, objects of a different shape but of the same volume increase by the same amount the volume of liquid in the container which contains them.

B. If two objects of the same weight but of different material will occupy the same volume. Thus the level of the water in the container will not increase at all.

3. Hypothesis.

1. The students report in writing that the objects with the same volume, regardless of their shape, will occupy the same amount of space, hence their volume does not change.

2. The students report in writing two objects of the same weight but of different material occupy the same volume. .

4. Theoretical framework

Piaget [2] distinguished two types of mathematical thought: one is figurative and refers to the ability to see things statically as and entirety, the other is operative and is related to intellectual is related to intellectual transformations, Koleza refers to a particularly worthwhile theoretical text in which Sfard[9] attempts επιχειρεί μια to put together the former dichotomies, proving that the learning process and the solution of problems in mathematics recommends itself to interaction of operative and constructive perceptions of the same concepts. The important thing is that in mathematics one can handle concepts through their representations and can record their attributes without the need of philosophical questions regarding the existence of mathematical objects.

Representations of mathematical objects are mostly semiotic representations**.** Semiotic representations are those which are expressed by the use of signes, (pronounced in natural language, algebraic models, graphic representations geometric diagrams) and are the means which are allocated to the individual in order to externalize intellectual. Consequently they are dependent on intellectual representations and serve no further than the necessity of communication.

The ability to handle mathematical objects is directly dependent on the semiotic system that is being used. Mathematical workings cannot take place except through semiotic representations. They cannot take place through intellectual representations [10].

Piaget (1968) considers representations as a substitute for the objects which they represent when those objects are not present. Considering this in our own work we have still to add that the term "representation" according to Von Glasersfeld (1987) [7] can possibly refer to: (a) pictorial representations, (b) symbols and (c) intellectual representations. Both the first (a) and the second (b) refer to external representations while the third (c) refers to internal representations. Von Glaserfeld Maintains that intellectual representations have a dynamic character since they are not simple registrations which are recalled to the mind of the students as if placed there in storage but consists of processes which are dynamically activated (Von Glasersfeld, 1987) [7]. The reports of Roth and McGinn (1998) [11]are on the same wavelength for the concepts of representations which are concentrated on the concept of registration. Registrations, because of their nature, are "social objects" (Roth & McGinn, 1998, p. 37)[11] with the idea that they can be communicated to many individuals. Duval (Duval 2005) [8] maintains that a concept needs to be offered to the student with multiple representations. This helps him to be able to describe fully the concept or the structure of the concept. With the use of multiple representations which refer to the same concept students can comprehend the common attributes of different representations and develop, since the structural relationships between situations which differ in their exterior characteristics is recognized. Thus the student can better approach mathematical knowledge (Greer & Harel 1998) [12]. Duval (Duval 1987)[8] had formerly maintained that each semiotic field has different possibilities.

Coming once more to the topic of volume we have to add that after Piaget there was important research done by Gelman (Gelman, 1969)[13] and McGarrigle and Donaldson (McGarrigle & Donaldson, 1974)[14].The basic point of this research was on how students conceive changes in volume.

The comparative elements of the liquid and solid maintenance of volume do not disagree with the statement that the concept of liquid maintenance of volume exists in children at an earlier stage than the corresponding concept regarding solids. Such a level of the solid maintenance of volume (in 71.7%) does not appear until the end of the 9th year (Shayer & Adey, 1981[15] [16][17]Shayer,1992). The maintenance of the liquid form of volume exists after the age of five (81.7%).

It is pointed out that the concept "maintenance of volume" is used if, for example, a liquid which exists in a short but wide container is emptied into a long, narrow container, then the child cannot understand that the quantity of liquid does not increase, because he sees the elongation in isolation and does not have the ability to compare it with reduction in the size of the perimeter of the container (Piaget &Inhelder, 1974) [1]. These studies prove that thinking on liquid maintenance of volume enters the repertory of a child more quickly than things which have to do with the maintenance of solid volume. McGarrigle and Donaldson attributes a large part of this inability to "a clear linguistic cause." He maintains that that a child takes language into consideration based on contextual elements. These contextual clues are considered irrelevant by adults (McGarrigle and Donaldson 1974)[14].

This inability which students have is due to the fact that students think as they describe the phenomenon. So the student understands what he is seeing at the first glance, without going on to further analysis of process and change as an adult would do. Gelman maintains that children can be taught not to let their judgment be distracted only by what they see. If a student does not examine different parameters then he will give a wrong answer. Thus it would be good if students, before they express themselves verbally, could examine things which are the consequence of a change which, at first glance, they did not perceive as a total change (Gelman, 1969 &Twidle, 2006) [13].

McGarrigle did experiments with bodies of different volume. The reason for these experimental methods which he developed was the checking of children's different perceptions in the maintenance of volume (McGarrigle & Donaldson 1974)[14].

Donaldson maintains that for all the attempts that took place to teach the students to deepen their thinking about what they can see, a significant number of children would continue to fail to prove that the volume of a liquid remains the same even if the container is changed. He compressed a ball of clay to make it greater in diameter and then posed the same question. Only older students understand that in the end the clay is the same. SimilarlySmedslund compressed the clay and put it in water to check if the volume of the play changed. The students were able to observe if the level of water increased. Smedslund repeated the experiment taking away a piece of the clay before putting it in the water. He observed that only children above a certain age who had more developed reasoning could organize rationally and was justifications that the volume of the body in this phase was different (Demetriou, A., Shayer, M. &Efklides, A. (1992) [15].

Rowell and Dawson make significant reference to the ideas of the students in different scientific experiments which they carry out. More specifically they create a process which includes two phases. In this process they used two spheres which they immerse in cylindrical containers. These spheres were made of plasticene. In the first phase of the experiment one of the two spheres was changed in shape. Both spheres were placed outside two cylindrical containers full of water. The students were asked to say if the level of water would be different if the sphere of the same weight but of different shape was placed in the second tube. Further, they were asked to explain the relationship of the volume of liquid which changed if a small piece of plasticene was removed from the sphere and replaced by the same volume of lead, what would happen to the volume of liquid in the tube [16], [17] [18].

5. Methodology of the Research

58 students from three sections of the third year of primary school in a provincial island city took part in the research in the school year 2006-2007 in the framework of the teaching of mathematics in the third year of Greek primary school.

The pupils were given worksheets, while the observations of the class were recorded on video. As regards the thematic axis of the questions, A basic concern was that suitable conditions should be created for the students to write on paper (worksheets) the reasons to support an opinion. Of interest to the researchers were the practices used by the pupil to solve the problems.

Regarding the problems which were set for the pupils, in this task the problems that were sent were simply **descriptive pictorial representations** related to weight and volume. Then the students were called upon to judge **whether or not there were any changes in these pictorial representations**i.e. how much the volume of a liquid increased or decreased, or if it stayed the same.

Care was taken that the interpretations which were given for the observed phenomena emanated from the pupils' experience. For the above reason specific experiments that the pupils could imagine were chosen.

6. Experimental Area

The pupils were asked to draw a container full of water. They also drew an object. The pupils were then asked to draw the same container again but this time in which they had placed the object.

It was observed that the students designed a new container with a higher level of water than in the first drawing.

Then, after any queries they might have had had been answered verbally, they were given the following two questions in writing

1st question:

The students were asked to determine the point to which the level of the water would rise if 10 iron boxes (in the shape of cubes) were placed in the container.

In an identical container (container B) we threw another 10 spheres. The pupils were asked to say if the water level would be more or less. The ball have the same volume as the 10 iron cubes. It was clarified to the people's that if there are also was positive (i.e. the level would rise) they should determine approximately how far the water level would rise. Into a third container were placed in long narrow cylindrical objects in the shape of rods. These objects also had the same volume as in the former cases. The pupils were again asked to determine the level of water in relation to the first and second containers. They were asked to justify their answers. Otherwise to write that there would not be a greater rise in the level of liquid in relation to container A or B.

Answers:

We're able to distinguish two kinds of answer**:**

"Correct" answers thus:

- These pupils has acquired the thinking that the objects irrespective of their shape would take up the same if they have the same volume hence the volume of water which would also go up in the three situations would be the same .

"Wrong" answers thus:

- Pupils who believe that the objects will increase the volume of a liquid differently according to their shape. So it was ascertained that "volume" as the concept has not been comprehended. The majority of the erroneous answers in this category consider that bodies in the form of cubes Increase the level of the water to a higher level compared to the others. More specifically the students believe that the long thin objects increase the level of water the objects occupy by less than do objects in the shape of spheres, while the objects in the shape of cubes have greater volume than the spheres or rods (Figure 2a & 2b)
- Despite the fact that in the announcement it was expressly declared that the bodies would be immersed in the container there were students who believed that there are objects which they could not put into the container, because their shape did not allow it. The reason is that in the minds of the pupils there is an erroneous picture to be considered that the boxes (the cubic objects) cannot be immersed because of their shape (Figure 3).
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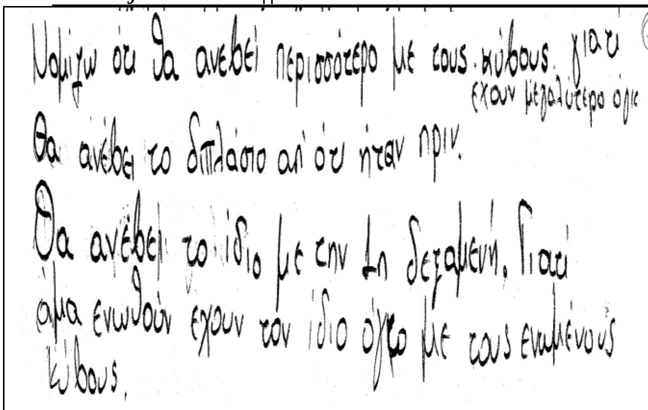


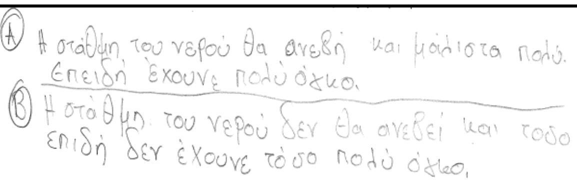


Figure 2a: I think that it will go up more with the cubes because they have greater volume. It will go up to double what it was before. It will go up the same in the 1st container. Because when they are connected they have the same volume as the linked cubes.

Figure 2b: In the A container the level of water went up a lot because it has a large volume (the cube).
In the B container the level of water will not be as much because they do not have as much volume.

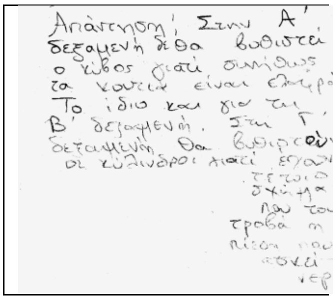


Figure 3: In the A container they cube will not be immersed because usually cubes are light. It will be the same for the B container.

Table 1 – Grouping of the pupils' written answers/ explanations

		Pupils	Percentage
1	Regardless of shape or weight, objects with the same volume will raise the water level by the same amount.	99	78,57%
2	Depending on their material, objects will increase the volume of water differently	17	13,49%
3	Some objects cannot be immersed in water because they float on account of their shape.	10	7,94%

2nd Question

In the second question the pupils were asked to say how much the level of water would go up if approximately 10kg of iron were placed in the container and how much it would go up if 10kg of a ceramic material were placed in the container. Verbally it was clarified to them as follows:

- That the two materials would go into the same container and that whatever estimate they recorded would be an
- The reason for this question was to ascertain comparatively with two different materials, what difference there would be in the elevation of the water level in relation to the type of material which was introduced into the container.

Some of the answers which we consider to be "**correct**":

- We do not know the volume of the iron. In order to recognize how much the volume of the island would go up we would need to know the volume of the 10kg of iron. For the ceramic material the students answered similarly " we do not know the volume of the ceramic material (for 10kg) in order to predict how much the level of liquid would go up"
- There were even students who were puzzled as to whether the iron and the surrounding material had the same volume, giving the answer that we would have the same result if the materials had the same volume (Figure 4a and 4b).

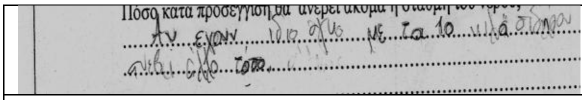
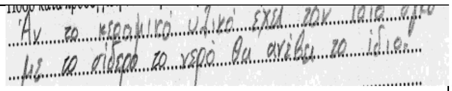
	

Figure 4aIf it had the same volume (the ceramic material) as the ten kilos of iron, it would go up by another so much..	***Figure 4b*** *If the ceramic material had the same volume as the iron it would go up a lot.*

We considered as "wrong" answers those which the pupils gave on general lines, which we have groped as follows:

7. Answers

The concepts of weight and volume are less specific but those which are realized at the age of eight or nine(4th class) the pupils cannot explain sufficiently, or, for the most part, even understand that volume and weight are two different sizes (Picture 5) Some pupils answer that:
-the weight makes the water rise in every situation.
-as much as the weight increases in the water, the volume of water increases by the same amount.

Figure 5: The water level will go up more because the ceramic material is lighter than the iron

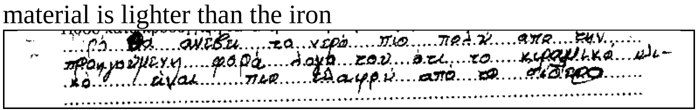
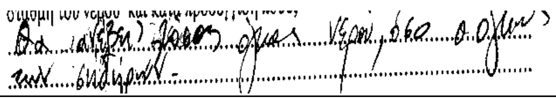
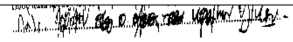


Figure 6: The volume of water will go up as much as the volume of iron. The same few will say that again the water would go up as much as the volume of the ceramic material .

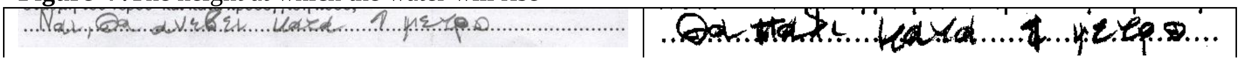
The volume of the water for the ceramic material will go up as much as the volume of the iron	*Again the same volume for the ceramic as with the iron*
	

B. Attempts at Fortuitous Quantification:

We also observed that the pupils tried to quantify fortuitously.

Thus a large number of Few pills although they were not given the level and the volume in the container recorded a number in their answers (Figure 7) .

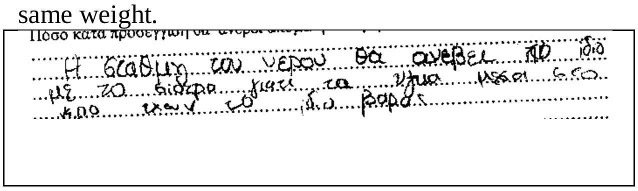
Figure 7: The height at which the water will rise



C. Different materials in the water have the same weight

There are students who believe that the materials have the same weight in the water and thus they answer erroneously. (Picture 8):

Figure 8: The level of the water will go up the same as the iron because the materials in the water have the same weight.



D. Answers without any justification:

There was another group of pupils who did not make any attempts to justify their thinking. They answered in both questions simply that will water level would go up (Figure 9)

In the following table the erroneous answers of the pupils are summarized.

Figure 9: The level of water will go up

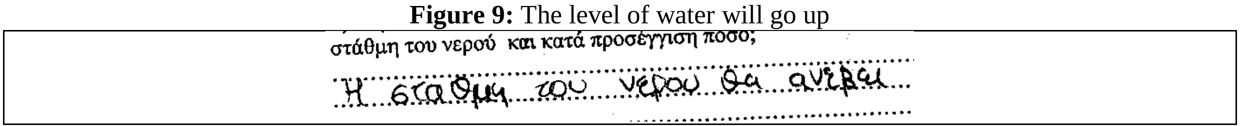


Figure 10: Ranking of incorrect questions

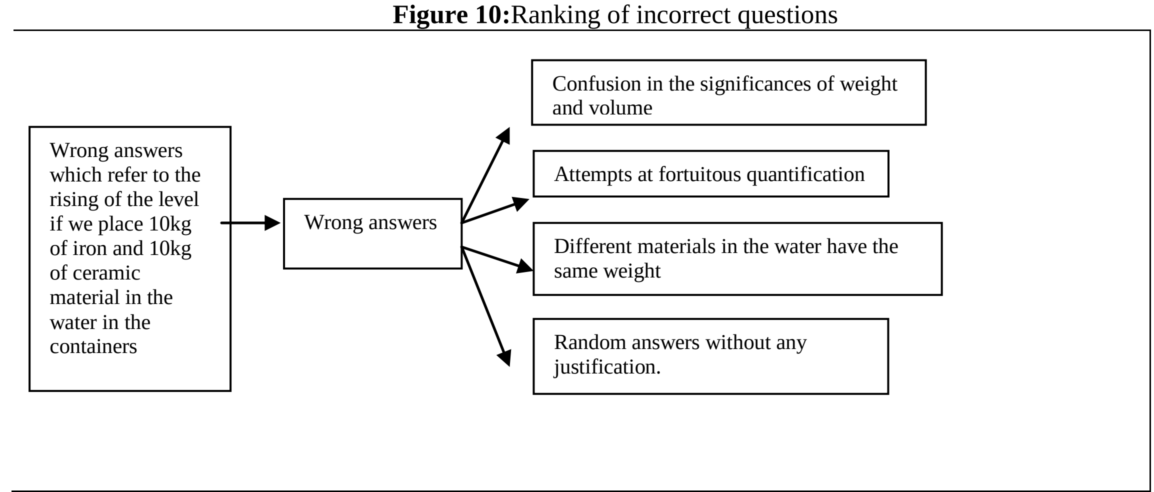


	Table 2. Grouping of Pupils' Answers	**Total number of Pupils**	**Percentage of Pupils**
1	Bodies of the same weight but different volume alter the surrounding area differently from a body immersed in water.	43	34,13%
2	Volume is determined by *the shape* of the bodies.	20	15,87%
3	Confusion of the concepts of weight and volume	47	37,80%
4	Different materials in the water have the same volume if they have the same weight	4	3,17%
5	Fortuitous answers with no justification, or no answer	14	11,11%

8. Conclusions

During the analysis of the pupils' answers we attempted to take out conclusions if they could cause regrouping of the pupils responses. We observe that in general there is a confusion in the written answers concerning the changes in volume depending on the material of the body (iron, or ceramic) without the pupils examining other parameters , such as quantity, weight etc. This confusion seems to exist also in the relationship between the shape and volume of a body.

As far as the **first question** is concerned we observed that there are pupils who can carry out hypothetical, inductive actions, and pupils whose thinking has not reached the level of making such hypothetical inductive action.

In the first category i.e. those who can carry out hypothetical, inductive actions, the pupils could connect two sizes such as volume and shape, generalizing their conclusions. These people's had advanced to the forming of specific intellectual actions (classes, relationships, space etc.)

Students whose thinking has not reached the level of carrying out hypothetical inductive actions. With these pupils it was observed that if they were told that the volume of the bodies was the same, they confused the concept of volume with that of weight, considering that for the ceramic material and the iron to have the same weight the value of the ceramic material must be greater. One might think that if the pupils had knowledge of density they would be able to grasp these concepts more correctly. But it's impossible for this to be understood by pupils in the fourth year of primary school, since even older pupils like those in Years 5 & 6 primary school and in the first year of junior secondary have difficulty in understanding the concept of density. These difficulties are due to the fact that density is not a quantity that can be observed or measured directly[19]. Density results as the reason for two other quantities of mass and volume [19],[11] concepts which exists in the problems that are set in this work, but they continue and maintain the children's misconceptions about material, with the results that they cannot differentiate concepts of weight and density [1]. This still fits in with the opinion of Hewson where density is related to the hardness of the object, so in our case with the hardness of the ceramic material. The ceramic material has a lesser density but a greater volume [20].

That the pupils gave which seemed to be important*:* that the pupils do not know what volume and weight means and have difficulty in separating these concepts. They believe that they have a reverse relationship dependent on an equally unknown concept (this concept in all probability is that of density referred to above. They also discern that the level of water goes up differently for different bodies or volumes.

As regards the **second question**, the reverse was attempted. That is to say that few bills were asked to describe what would happen if into the container replaced the same weight of ceramic material and the same weight of iron.

The immediate superiority of the synopsis of the results can be repeated here too where one can distinguish more types of error. Observing figure 8 Where the forms of **erroneous answers** are grouped it can be seen that many pupils come to conclusions without a methodical investigation. Thus it may be discerned that they do not have Έτσιδιακρίνεταιότιδενέχουν a starting point in their thinking, a fixed point of reference to which it is possible to return whenever it is needed. This is probably due to the fact that the pupils cannot recognize what is included in the knowledge of the basic elements which determine the concept of volume or of weight.Volume and weight are concepts. As we know, concepts are the basis of human thinking and communication. The understanding of any subject whatsoever is not possible if fundamental concepts are not acquired by the pupils. According to this process, learning is possible because we are able to reveal common attributes in different kinds of experience which are “stored” in the memory for future use.

So the pupils do not have the ability to recognize relationships which connect volume with weight and weight with volume or shape with volume. Nor can the rate the aforementioned concepts so that even if they arrive at the solution to the problem they cannot organize their thinking into a logical serious to pass from one concept to another (translation).

Thus the translational faculty of the individual can be characterized as rather poor since it is not distinguished by the possession of the required or suitable knowledge. In addition the pupil is unable to develop a system for taking in and processing information through his own experiences.

9. Future invstigation. Formation of teaching based on previous Evaluation.

In this work we just the onset of the students based on our own conceptual elements. So having created this knowledge of the concepts and functions of a child in the future work we must form our teaching in such a way us to make the above concepts understandable as shown in the above examples.

10. Limitation

Although we believe that the problems were satisfactory for the conclusions that we came to, the use of a limited number of experimental elements can reduce the reliability of the research. Consequently, it might be wise to look at any conclusions or generalizations with prudence, and to repeat similar tests in the future.

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